

EXAMINING CRASH VARIABLE BASED ON COLLISION TYPE FOR PREDICTING CRASH SEVERITY ON URBAN HIGHWAYS

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Abstract: Transportation facilities are the backbone for any economic growth in country's development. However, presently highways are the biggest threat to the world as they account for the largest number of road crashes. As a highway engineer emphasis should be made on reducing the occurrence of road crashes and understanding the causative factors. It is also important to understand the relationships between the severities of a road crash in the context with collision types. Several studies were carried on factors that influence on severity of crashes. The present paper attempts to explore the various factors associated with crash prediction on Indian highway. The study was conducted on National Highway NH-44 in Hyderabad city of Telangana State. Multinomial logit model was used for assessing the variables that influence the crash severity. The model reveals that vehicle type plays a major role in increasing the severity of the crash on highways. The model will be useful for the highway planner while improving the road section reducing the number of crashes.

Keywords: crash, collision, highway, multinomial logit.

1. Introduction and Review of Literature

An in-depth understanding of the random nature of crash process is one of the most interesting as well as essential aspects of traffic safety analysis. These will be widely used in practical implications for providing a safe transportation facility. In India, the highest number of fatal road crashes occurs on National Highways (NH) as 32.6% followed by State Highways (SH) as 27.8% fatality (NCRB, 2014). Understanding 'the random nature of crash process' on highway segments is of utmost importance as this can pave way for timely interventions to save precious lives. In this context a study was

conducted on NH 44 from Suchitra to Tupran for about 40kms in length in Hyderabad city of Telangana State. It is observed that 802 crashes have occurred during the last 5 years period on this highway segment. This paper explains the causative factors that influence the collision pattern for occurrence of crash. Several studies were carried during a decade period for developing crash prediction models through statistical tools.

Shankar *et al.*, (1995) used a negative binomial model to estimate crash frequencies and concluded that separate regression models shall be used for different types of crashes which will explain much better than

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a single crash model. Khattak *et al.*, (1998) estimated binary probit models to analyse severity of crashes during adverse weather conditions and identified that the frequency of crashes is higher during adverse weather but their severity is low. Dissanayake and Lu (2002) explained that a set of binary logistic regression models for single-vehicle crash data and identified important factors that result in higher crashes and severe injuries. Abdel-Aty (2003) analysed the severity of crashes at different locations using ordered probit models and identified variables that significantly cause severe injuries for that particular location. Donnell and Mason (2004) developed logistic regression models for cross-median and median crashes by considering the severity of a crash as ordinal response and nominal response respectively. Lord *et al.*, (2005) utilized zero-inflated Poisson and negative models on crash data and concluded that these are the most reliable models when crash count data has a large number of samples that are 'perfectly safe'. Lord (2006) fitted Poisson-gamma models on crash data that has a low sample mean and small sample size and showed that they have a very serious effect on the estimated parameters of the model and reduce the reliability of the model. Qin *et al.*, (2006) investigated the relationship between crash occurrence and hourly volume by fitting binary regression models. He has concluded that the relationship between crash types and independents is non-linear in nature. Kim *et al.*, (2007) applied multilevel binomial logistic models in order to predict the probability of different crashes based on the hypothesis that the intersection crashes are hierarchical in nature. Huang *et al.*, (2008) established a Bayesian hierarchical binomial logistic model and identified that light weight vehicles cause major injuries. Quddus *et al.*, (2009) used ordered-response models to

observe the effect of traffic congestion on crash severity and found that there is no relationship between them. Geedipally *et al.*, (2010) developed multinomial logit models for different crash types and found that predicting crash counts by specific crash type is the best among the existing methods. Ramesh and Kumar (2011) explained the current road safety level in Hyderabad city. Multinomial regression analysis was carried for analysing accident prediction model. Black spots were also identified using statistical methods. Yasmin and Eluru (2013) compared various crash data models and concluded that mixed generalized ordered logit competes with mixed multinomial logit in modelling crash type data. Ye and Lord (2014) examined the effect of sample size and concluded that ordered probit model can work efficiently for smaller sample sizes. De Luca (2015) worked on accident data using multivariate analysis and artificial neural networks and concluded that multivariate analysis is more efficient in the analysis of blackspots. Niveditha *et al.*, (2015) explained that logit models are suitable for prediction of the crash. The results explain that shoulder condition is an affecting factor for the occurrence of non-fatal crashes in the city.

Most of the literature was on homogenous traffic condition. A few studies in India, particularly for our city were conducted in analysing the causative factors of crash on highways. In the present work, multinomial logit model is used for understanding different crash types. Five characteristics were considered namely, causes of crashes, road curve radius, the type of vehicle that caused the crash, the pavement condition during which the crash has occurred and time of crash. It is hypothesized that these characteristics influence the severity of the crash.

2. Methodology

Multinomial Logit (MNL) Model was used in predicting the impact of crashes on the basis of collision type. This is accomplished by establishing a relationship between the severity of the crash and crash variables. Table 1 provides the crash variables and its code used for model development.

Hosmer and Lemeshow (2000) mathematical expressions were used for model development, where Y is considered as severity of the crash and x_1, x_2, x_3, x_4, x_5 as crash variables. Y consists of 4 levels namely non injury, minor injury, grievous injury and fatal injury. We have three logit functions as $logit_1, logit_2, logit_3$, expressed mathematically as below.

$$logit_1 = \ln \left[\frac{P(Y = 1|x)}{P(Y = 0|x)} \right] = \beta_{10} + \beta_{11}x_1 + \beta_{12}x_2 + \beta_{13}x_3 + \beta_{14}x_4 + \beta_{15}x_5 \quad (1)$$

$$logit_2 = \ln \left[\frac{P(Y = 2|x)}{P(Y = 0|x)} \right] = \beta_{20} + \beta_{21}x_1 + \beta_{22}x_2 + \beta_{23}x_3 + \beta_{24}x_4 + \beta_{25}x_5 \quad (2)$$

$$logit_3 = \ln \left[\frac{P(Y = 3|x)}{P(Y = 0|x)} \right] = \beta_{30} + \beta_{31}x_1 + \beta_{32}x_2 + \beta_{33}x_3 + \beta_{34}x_4 + \beta_{35}x_5 \quad (3)$$

Where: $logit_1$ represents a logit function for Minor Injury versus Non-Injury, $logit_2$ for Grievous Injury versus Non-Injury and $logit_3$ for Fatal Injury versus Non-Injury. $\beta_{10}, \beta_{20}, \beta_{30}$ represent the intercepts and the remaining β 's represent the coefficients of the characteristics.

The odds ratio is the exponent of the coefficient obtained for characteristic in the logit model. When odds ratio < 1, there is a higher chance for occurrence of non-injury. If the odds ratio > 1, it implies that an increase in the level of crash variable/characteristic leads to a higher chance of

occurrence of crash severity, i.e. minor injury for $logit_1$, grievous injury for $logit_2$ and fatal injury for $logit_3$. MNL model is tested for its significance using *likelihood ratio*. The statistical analysis was implemented by multinom() in nnet package of R 3.3.1.

3. Data Collection

Crash data was collected on National Highway 44 (NH-44) for 5yrs period (2011–16) along the study stretch. The length of study stretch is 40 km from Suchitra X-road to Tupran of Telangana State and is shown in Fig 1.

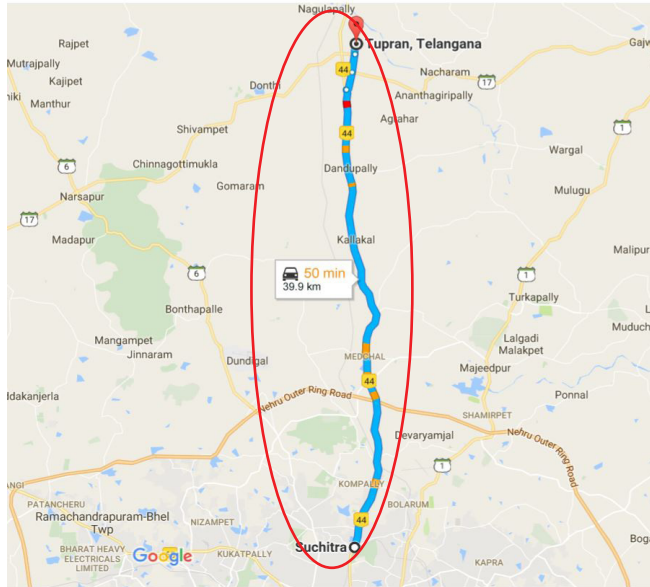


Fig. 1.
Study Location for Crash Analysis

It is observed from the data that crash is influencing the following collision types:

- a) Head-on Collision
- b) Rear-end Collision
- c) Side swipe Collision.

The true impact of collision type shall be used in assessing the severity of the crash. The severity of crash in the study location is observed in four levels (“Severity of Crash/ Code”): Non-Injury/0, Minor Injury/1, Grievous Injury/2, Fatal Injury/3.

Table 1
Variables Influencing Severity of Crash

S.No.	Crash characteristic	Codes
1	Causes	1 = Drunken Driving 2 = Drowsing 3 = Over speeding 4 = Vehicle out of control 5 = Fault of two-wheeler/pedestrian 6 = Defect in condition of vehicle condition
2	Road curve radius	1 = Straight Road 2 = Slight Curve - $750 \leq R \leq 900$ m 3 = Sharp Curve - $300 \leq R \leq 400$ m
3	Vehicle responsible	1 = Light weight 2 = Heavy weight
4	Pavement Condition	1 = Dry Pavement 2 = Wet Pavement (Rain)
5	Day/Night	1 = Day 2 = Night

Crash characteristics and severity of crash for each collision type is shown in Table 2.

Table 2
Summary of Crash Characteristics

Characteristics		Type of Collision					
		Head-on Collision		Rear-end Collision		Side swipe collision	
		Count	%	Count	%	Count	%
Severity of Crash	Non-Injury	7	4.8%	17	4.6%	11	22.9%
	Minor injury	24	16.3%	94	25.4%	20	41.7%
	Grievous Injury	43	29.3%	180	48.6%	12	25.0%
	Fatal Injury	73	49.6%	79	21.4%	5	10.4%
Causes	Drunken Driving	2	1.4%	15	4.2%	1	2.1%
	Drowning	1	0.7%	2	0.5%	0	0.0%
	Over speeding	102	69.4%	291	78.6%	39	81.2%
	Vehicle out of control	19	12.9%	27	7.3%	4	8.3%
	Fault of two-wheeler/pedestrian	22	15.0%	32	8.6%	2	4.2%
	Defect in condition of vehicle condition	1	0.6%	3	0.8%	2	4.2%
Road curve radius	Straight Road	128	87.1%	318	85.9%	40	83.3%
	Slight Curve	8	5.4%	30	8.2%	5	10.4%
	Sharp Curve	11	7.5%	22	5.9%	3	6.3%
Vehicle Responsible	Light weight	100	68.0%	213	57.6%	23	47.9%
	Heavy weight	47	32.0%	157	42.4%	25	52.1%
Pavement Condition	Dry	44	29.9%	122	33.0%	14	29.2%
	Wet	103	70.1%	248	67%	34	70.9%
Day/Night	Day	72	49.0%	199	53.8%	26	54.2%
	Night	75	51.0%	171	46.2%	22	45.8%

4. Model Interpretation and Findings

Four Multinomial logistic models are developed to predict the severity of the crash. The first MNL explains on total Collisions and the remaining three models explain on Head-on Collision, Rear-end Collision and Sideswipe Collision respectively. The coefficients of characteristics along with their Standard Errors (SE) and odds ratios are computed for each model. A Z-statistic with its p-value is observed for estimated coefficients which explain the influence of crash variables in the model. These are presented in the tables 3-7.

Table 3 interprets that for *logit*, the coefficients of causes, road curve radius and pavement condition are positive, implying that a unit increase in these characteristics is associated with an increase in the chance of injury being a minor injury. The coefficients of the vehicle responsible and day/night are negative, indicating that a unit decrease in these characteristics results in an increase in the chance of minor injury. The odds ratio is greater than 1 for causes, road curve radius and pavement condition; further if the values of these characteristics increase, there is a higher chance of minor injury. The odds ratio is less than 1 for vehicle responsible

and day/night indicating a higher chance for non-injury when the values of these characteristics increase.

In case of *logit*₂ two characteristics namely causes and pavement condition have positive coefficients implying that a unit increase in these characteristics results in an increase in the chance of grievous injury. A unit decrease in road curve radius, vehicle responsible and day/night leads to an increase in the chance of grievous injury. The odds ratios of causes and pavement condition are greater than 1 implying that a grievous injury has a higher chance of occurrence as the values of these characteristics increase. The odds ratios for road curve radius, vehicle responsible and day/night are less than 1 indicating that the chance of occurrence of non-injury increases as the values of these characteristics increases.

In *logit*₃, all characteristics except vehicle responsible have positive coefficients indicating that a unit increase in each of these characteristics implies a corresponding increase in the chance of fatal injury. The characteristic vehicle responsible has a negative coefficient implying that a unit increase in this characteristic decreases the chance of fatal injury. The odds ratio of causes, road curve radius, pavement condition and day/night is greater than 1 depicting that an increase in the values of these characteristics results in a higher chance for occurrence of fatal injury. The odds ratio of vehicle responsible is less than 1 which means that increase in the value of this characteristic leads to a higher chance of non-injury. Tables 4, 5 and 6 interpret for head-on, rear-end and side swipe collision types with the characteristics influencing severity of crash.

Table 3

Estimated Coefficients, Standard Errors and Their Odds Ratios for Total Collisions MNL Model

	Characteristics	Coefficients	SE	Z	p-value	odds ratio
<i>logit</i> ₁	(Intercept)	5.2000	1.6541	3.1437	0.0017	
	Causes	0.4178	0.2658	1.5719	0.1160	1.5186
	Road curve radius	0.1922	0.2913	0.6596	0.5095	1.2119
	Vehicle Responsible	-2.6710	0.6297	-4.2416	0.0000*	0.0692
	Pavement condition	0.0719	0.2164	0.3325	0.7395	1.0746
	Day/Night	-0.5704	0.3958	-1.4411	0.1496	0.5653
<i>logit</i> ₂	(Intercept)	6.8400	1.6261	4.2065	0.0000	
	Causes	0.3771	0.2602	1.4489	0.1474	1.4580
	Road curve radius	-0.0042	0.2919	-0.0143	0.9886	0.9958
	Vehicle Responsible	-3.1532	0.6217	-5.0721	0.0000*	0.0427
	Pavement condition	0.1228	0.2101	0.5846	0.5588	1.1307
	Day/Night	-0.6590	0.3851	-1.7114	0.0870	0.5174
<i>logit</i> ₃	(Intercept)	4.9150	1.6462	2.9857	0.0028	
	Causes	0.4085	0.2636	1.5499	0.1212	1.5046
	Road curve radius	0.0402	0.2954	0.1359	0.8919	1.0410
	Vehicle Responsible	-2.8690	0.6268	-4.5772	0.0000*	0.0568
	Pavement condition	0.0927	0.2137	0.4337	0.6645	1.0971
	Day/Night	0.0270	0.3933	0.0687	0.9452	1.0274

* indicates a significant characteristic

Table 4*Estimated Coefficients, Standard Errors and Their Odds Ratios for Head-on Collision MNL Model*

	Characteristics	Coefficients	SE	Z	p-value	odds ratio
<i>logit₁</i>	(Intercept)	22.9207	1.4946	15.3357	0.0000	
	Causes	1.1562	1.1556	1.0005	0.3170	3.1778
	Road curve radius	0.0376	0.5668	0.0663	0.9472	1.0383
	Vehicle Responsible	-12.5530	1.8545	-6.7691	0.0000*	0.0000
	Pavement condition	0.0913	0.5560	0.1643	0.8695	1.0957
	Day Night	-1.2169	1.0094	-1.2055	0.2280	0.2962
<i>logit₂</i>	(Intercept)	22.8150	1.3140	17.3634	0.0000	
	Causes	1.2176	1.1400	1.0681	0.2855	3.3791
	Road curve radius	-0.0968	0.5549	-0.1745	0.8615	0.9077
	Vehicle Responsible	-12.4542	1.8444	-6.7523	0.0000*	0.0000
	Pavement condition	-0.0495	0.5266	-0.0940	0.9251	0.9517
	Day Night	-0.6088	0.9497	-0.6410	0.5215	0.5440
<i>logit₃</i>	(Intercept)	21.7772	1.2442	17.5024	0.0000	
	Causes	1.2411	1.1337	1.0947	0.2736	3.4595
	Road curve radius	0.0548	0.5342	0.1026	0.9183	1.0564
	Vehicle Responsible	-12.5286	1.8492	-6.7752	0.0000*	0.0000
	Pavement condition	0.0588	0.5150	0.1141	0.9092	1.0605
	Day Night	0.1499	0.9308	0.1611	0.8720	1.1618

“*” indicates a significant characteristic

Table 5*Estimated Coefficients, Standard Errors and Their Odds Ratios for Rear-end Collision MNL Model*

	Characteristics	Coefficients	SE	Z	p-value	odds ratio
<i>logit₁</i>	(Intercept)	5.7870	2.1843	2.6494	0.0081	
	Causes	0.3896	0.3563	1.0934	0.2742	1.4763
	Road curve radius	0.0437	0.3782	0.1155	0.9080	1.0447
	Vehicle Responsible	-1.8816	0.7986	-2.3561	0.0185*	0.1523
	Pavement condition	-0.1400	0.2970	-0.4714	0.6374	0.8694
	Day/Night	-1.1606	0.6209	-1.8692	0.0616	0.3133
<i>logit₂</i>	(Intercept)	7.8845	2.1546	3.6593	0.0003	
	Causes	0.2510	0.3513	0.7143	0.4751	1.2853
	Road curve radius	-0.2312	0.3819	-0.6055	0.5448	0.7936
	Vehicle Responsible	-2.6680	0.7859	-3.3947	0.0007*	0.0694
	Pavement condition	0.0781	0.2906	0.2689	0.7880	1.0813
	Day/Night	-1.1640	0.6099	-1.9085	0.0563	0.3122
<i>logit₃</i>	(Intercept)	5.6525	2.2041	2.5645	0.0103	
	Causes	0.0244	0.3633	0.0671	0.9465	1.0247
	Road curve radius	-0.1232	0.3925	-0.3138	0.7537	0.8841
	Vehicle Responsible	-1.8864	0.8007	-2.3558	0.0185*	0.1516
	Pavement condition	0.0820	0.2991	0.2740	0.7841	1.0854
	Day/Night	-0.5758	0.6285	-0.9160	0.3596	0.5623

“*” indicates a significant characteristic

Table 6

Estimated Coefficients, Standard Errors and Their Odds Ratios for Side Swipe Collision MNL Model

	Characteristics	Coefficients	SE	Z	p-value	odds ratio
<i>logit</i> ₁	(Intercept)	-9.3548	86.5357	-0.1081	0.9139	
	Causes	0.9952	0.9842	1.0112	0.3119	2.7053
	Road curve radius	10.1107	86.4541	0.1169	0.9069	24604.5200
	Vehicle Responsible	-3.4603	1.2762	-2.7113	0.0067*	0.0314
	Pavement condition	0.9638	0.7245	1.3302	0.1834	2.6217
<i>logit</i> ₂	Day/Night	0.1816	0.9764	0.1860	0.8524	1.1992
	(Intercept)	-7.5927	86.5382	-0.0877	0.9301	
	Causes	0.5874	1.0149	0.5788	0.5627	1.7994
	Road curve radius	10.0011	86.4540	0.1157	0.9079	22050.3200
	Vehicle Responsible	-2.6741	1.2801	-2.0890	0.0367*	0.0690
<i>logit</i> ₃	Pavement condition	0.3340	0.7119	0.4692	0.6390	1.3965
	Day/Night	-0.2347	0.9754	-0.2406	0.8098	0.7908
	(Intercept)	-11.3386	86.5894	-0.1309	0.8958	
	Causes	1.4736	1.1007	1.3388	0.1806	4.3650
	Road curve radius	9.4845	86.4566	0.1097	0.9126	13153.6200
	Vehicle Responsible	-2.9676	1.5017	-1.9761	0.0481*	0.0514
	Pavement condition	-0.2420	0.9219	-0.2625	0.7930	0.7851
	Day/Night	1.0776	1.3479	0.7995	0.4240	2.9375

‘’ indicates a significant characteristic*

The results interpret that the above characteristics influence the severity of the crash. The crash variable vehicle responsible is observed as the major factor influencing crash severity. For all collision types, four-wheeler vehicles and below are considered as

light weight vehicles and all vehicles above four-wheelers are considered as heavy weight vehicles in the analysis. It is observed from the results that light weight vehicles have a higher severity of crashes as opposed to heavy weight vehicles.

Table 7

Goodness-of-fit Statistics

Collision type	log-likelihood	Chi-Square	p-value
Total	-791.760	73.689	<0.0001
Head-on	-168.760	28.706	0.017
Rear-end	-432.840	48.983	<0.0001
Side swipe	-61.660	25.495	0.0437

The log-likelihood measure values for total collisions, head-on, rear-end and Sideswipe are significant at 5% level of significance which concludes that these models provides a good fit to the data.

5. Conclusions

The paper introduces a model-based mechanism to establish relationships between the nature of collisions and various characteristics, namely; causes of crashes, road curve radius, vehicle type, pavement conditions and time of the crash. The severity of crashes is considered to be the systematic component of the nature of collisions in this study. Multinomial logit models are developed to understand the effect of these characteristics on the level of severity of the crash.

Statistical analysis and model interpretations conclude that the type of vehicle responsible for the crash plays an important role in determining the level of crash severity. This holds good for all the types of collisions considered in the study.

MNL model serves as an aid to the policymakers in framing norms to ensure that roads transform to safer spaces. Placement of warning signs, reflective mirrors and improved infrastructure design will improve safety for road users. The above model helps in predicting the level of severity of the crash based on the observed characteristics. This will pave way for timely interventions to save precious lives.

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