THE DYNAMIC BEHAVIOUR OF GEARS WITH HIGH TRANSMISSION RATIO

Ivana Atanasovska[1], **Marija Vukšić Popović**[2], **Zorica Starčević**[3]
[1, 2, 3] Institute Kirilo Savić, Vojvode Stepe 51, Belgrade, Serbia

Received 16 January 2012; accepted 27 February 2012

Abstract: This paper describes the dynamic behaviour of helical gears with new standpoint for calculation of influence variables: mesh teeth stiffness, contact lines lengths and load distribution during mesh period. Nonlinear contact Finite Element Analysis and the new iteration procedure are used for calculation of meshed teeth deformations, stiffness and contact loads. The normal load distribution calculated with this procedure is used for evaluation of nonlinear dynamic analytical model of helical gears motion. Described investigation is especially important for gear pairs with high value of transmission ratio, often used in large transport machines. The presented models and results can be used for helical gears modeling when standard procedures don't cover the requirements.

Keywords: gears, transport machines, dynamics.

1. Introduction

The load distribution during helical gear meshing is characterised with variable number of teeth pairs in contact and with different contact line length for simultaneously meshed teeth pairs. The solution for load distribution in helical gear mesh can be obtained only with precise calculation of mesh teeth stiffness and contact lines lengths during mesh period. The mesh teeth stiffness is a parameter that changes during teeth pair mesh period and also along teeth pair contact line. The complex helical gears geometry and variable length and position of tooth contact lines during mesh period results in complex and nonlinear load distribution in gear mesh.

There are many researches concerned with gears dynamics. The authors have modelled and calculated dynamic behaviour of gears, but only for spur involute gears. Thus, some authors (Amabili and Rivola, 1997; Dimitrijevic et al., 2007; Parker et al., 2000) modelled pair of gears by two disks coupled with non-linear mesh stiffness and mesh damping. Others (Faggioni et al., 2011; Lakota and Stajnko, 2009) transform the gear model with two steps of freedom to gear model with one step of freedom by reduction to the line of action. Different authors have used in their researches one of these models. The aim of this paper is to give results for both of models simultaneously and to calculate the time defence functions of gear mesh stiffness, load distribution and dynamics of helical involutes gears.

2. Theoretical Gear Pair Dynamic Model

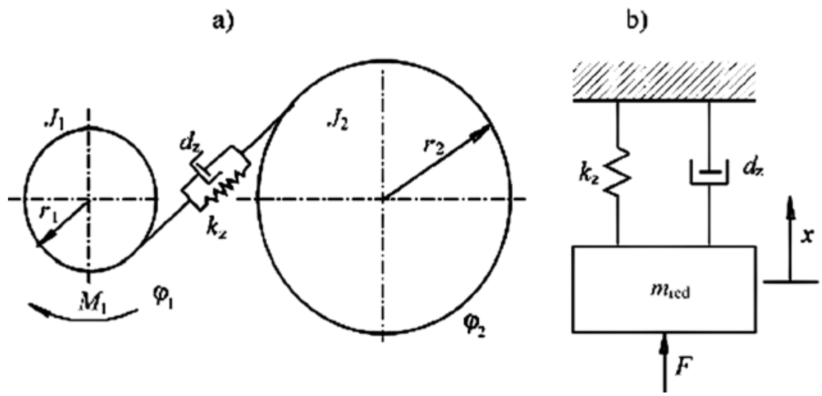
To describe the dynamic behaviour of helical gears, a pair of gears is simulated with two disks coupled with non-linear mesh stiffness

[1] Corresponding author: ivana.atanasovska@iks.rs

and mesh damping. One disk (driving gear) has radius R_1 and mass moment of inertia J_1 and the other (driven gear) has radius R_2 and mass moment of inertia J_2. The radii R_1 and R_2 correspond to the radii of the base circles of the two gears, respectively (Fig. 1a). The dynamic transmission error is the difference between the actual and ideal position of the driven gear and can be expressed as a linear displacement along the line of action (x). It is very important to calculate the $x(t)$ function (t - time) to predict gear noise and increase gear's life. Reduction to the line of action (Faggioni et al.,, 2011, Lakota and Stajnko, 2009), transforms the gear model II (model with two steps of freedom, Fig. 1a) to gear model I (model with one step of freedom, Fig. 1b).

$$m_{red}\ddot{x} + d_z(t)\dot{x} + k_z(t)x = F_n(t) \qquad (2)$$

In these equations: m_i (i = 1, 2) are equivalent masses of gears and m_{red} is reduced mass of gears, $k_z(t)$ is the gear mesh stiffness function, $d_z(t)$ is the gear mesh dumping function and $F_n(t)$ is the function of normal load distribution between the gear teeth in mesh, during gear meshing period. The mathematical models can be used for the analysis of the gears oscillation parameters, only if all functions are known. In this paper, the stiffness and load distribution are calculated using the true contact geometry of the helical gears as elastic bodies. The developed finite element model, (Atanasovska et al., 2007a; Atanasovska et al., 2009), is used. Damping value is assumed to be equal to 1 which excludes its influence



1a) Model II - with Two Steps of Freedom

1b) Model I - with One Step of Freedom

Fig. 1.
Dynamic Model of Gear Pair

Nonlinear dynamic analytical model of involute helical gears motion for model II can be given with equations (Eq. (1)):

$$m_1\ddot{x}_1 + d_z(t)(\dot{x}_1 - \dot{x}_2) + k_z(t)(x_1 - x_2) = F_n(t)$$
$$m_2\ddot{x}_2 + d_z(t)(\dot{x}_1 - \dot{x}_2) + k_z(t)(x_1 - x_2) = F_n(t) \qquad (1)$$

and for model I with Eq. (2):

on the gears oscillation characteristics, (Lakota and Stajnko, 2009). The calculation of nonlinear damping in dynamics of gear mesh is subject that requests widely access and research (Hortel and Skuderova, 2007).

3. Analytical Solution of Nonlinear Load Distribution for Helical Gears

The complex helical gears geometry and variable length and position of tooth contact lines during mesh period results in complex and nonlinear load distribution in gear mesh. Therefore, this task could be solved only by resolving the load distribution between simultaneously meshed teeth pairs and the load distribution along each of teeth pair contact line, at the same time. For mathematical definition of load distribution in helical gear mesh, the expanded procedure for load distribution over gear facewidth for involute spur gear (Atanasovska et al., 2007b) has been used.

For every moment (contact position P) during helical gears meshing period m tooth pairs are simultaneously in contact. System of integral equations, which consists of the contact equation and balance equation, can be defined for each i[th] of m simultaneously meshed tooth pairs for single contact position. This system can be presented in the following form:

$$\int_0^{B_i} q_i(z)\cdot K_i(z,u)dz = \Delta_i + F_{\beta i}(z) \qquad (3)$$

$$\int_0^{B_i} q_i(z)dz = F_{bni} \qquad (4)$$

Where: $q_i(z)$ – is function of unit load change along the i[th] tooth pair contact line, B_i – is length of i[th] teeth pair contact line for position P, $K_i(z,u)$ – is influence function, which defines relation between u (elastic deformation at one particular point on the contact pattern) and $q_i(z)dz$ (concentrated load at the same point), z – is coordinate of studied point along contact pattern, Δ_i – is total tooth pair deformation in the direction normal to tooth pair contact pattern, $F_{\beta i}(z)$ – is mesh initial misalignment (deviation between pinion tooth facewidth direction and wheel tooth facewidth direction when the gear pair is unloaded), F_{bni} – is total normal load value for i[th] tooth pair in mesh.

Systems of Eq. (3) and Eq. (4) for all m simultaneously meshed tooth pairs and equation of load balance (Eq. (5)):

$$F_{bn1} + F_{bn2} + \ldots + F_{bnm} = F_{bn} \qquad (5)$$

give system of (2m+1) equation for load distribution solution. It's very hard or almost impossible to determine real values for many factors and variables that have crucial influence on accurate form of the function $q_i(z)$, as well as on value of real tooth pair bearing pattern length B_i. Therefore, this system of integral equations can be solved only by numerical method usage with same simplification and assumptions.

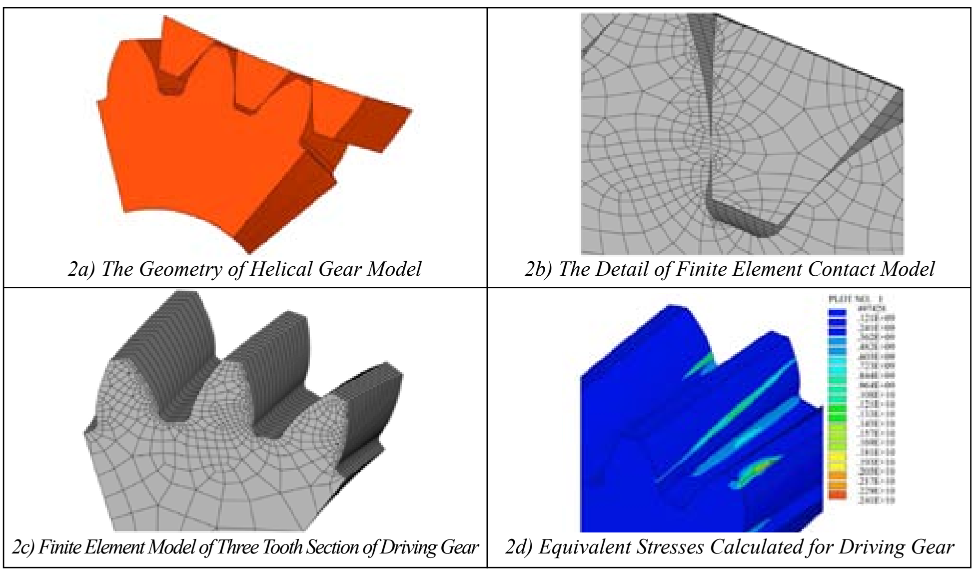
The discrete method is a simplification model chosen for solving the load distribution in gear mesh. The main difference between discrete method developed for spur gear pair, (Atanasovska et al., 2007b) and helical gear pair is in the number of tooth pairs that are simultaneously in mesh and in variable tooth pair bearing pattern length. The main principle of this method defines tooth pairs contact patterns like finale number of equal segments. Generally, a length of these segments is nearly a value of gear pair module. It is consider that the n_i is the number of segments on the i[th] tooth pair contact line, so the Eq. (5) takes the following form:

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i} q_{ij}\cdot B_{ij} = F_{bn} \qquad (6)$$

where q_{ij} – is normal unit load along the j^{th} segment of i^{th} tooth pair contact line; B_{ij} – is length of the segment. The numerical Finite Element Method is used for calculation of these values. In this way the nonlinear load distribution in helical gears mesh can be solved with the minimum assumptions and simplifications.

4. Numerical Methods and Calculation Procedures

The Finite Element Method (FEM) is the numerical method that gives the possibilities for developing of appropriate models in accordance with needs in modern researches of nonlinear tasks, (Amezketa et al., 2009; Andersson and Vedmar, 2003). This chapter of the paper describes modeling of helical gears by FEM. Appropriate analysis is performed by Atanasovska (Atanasovska et al., 2007a; Atanasovska et al., 2009) in order to select meshed gears model which is sufficiently economic and in same tame sufficiently geometrically accurate. The chosen FEM models each consist of three gear's teeth (Fig. 2). The special algorithm for tooth's involute profile drawing is developed and built in present FEM software to assure drawing of real involute flanks contact geometry. In addition, choice of optimal mesh size level is performed. Described FEM models are made for one particular gear pair with high value of transmission ratio, that enable us to perceive all potential problems during stress and strain calculations. The main characteristics of the gear pair are: number of teeth $z_1 = 20$, $z_2 = 96$; standard tooth involute profile, addendum modification coefficients $x_1 = 0.3$, $x_2 = 0.2$; face width b = 175 mm; module $m_n = 24$; pressure angle $a_n = 20°$; helix angle b = 15°; rotational wheel speed $n_2 = 4.1596$ min^{-1}; wheel torque $T_2 = 1264.4$ KN×m, material: steel with E = 206 000 N/mm^2; n = 0.3, pinion teeth inclination – right, wheel teeth inclination – left. So, for defined geometry characteristics and torque, the normal nominal load that this gear pair transmits is $F_{bn} = 1168.0354$ KN. The procedure gives the possibilities for monitoring of deformation and stress variables (Fig. 2d) during tooth pair meshing period.



2a) The Geometry of Helical Gear Model

2b) The Detail of Finite Element Contact Model

2c) Finite Element Model of Three Tooth Section of Driving Gear

2d) Equivalent Stresses Calculated for Driving Gear

Fig. 2.
The Helical Gear FEM Model

Because the tooth deformations in contact points depend of contact load values, the iteration procedure is used for solution of load distribution. In first step (iteration), total normal load for contact position P is divided on simultaneously meshed tooth pairs in proportion of appropriate length of tooth contact lines, which are scanned from FEM nonlinear contact solution. Therefore, the normal load on i^{th} meshed tooth pair is (Eq. (7)):

$$F_i = B_i \cdot F_{bn} / \sum_{i=1}^{m} B_i \qquad (7)$$

Then, tooth pair stiffness of j^{th} segment on i^{th} line of contact is calculated as ratio of unit normal load and total teeth deformations in the direction of line of contact scanned from FEM results. For the next iteration normal load is divided on tooth pairs in mesh in accordance with stiffness values, (Atanasovska et al., 2007b). The small permissive error enables confidential results.

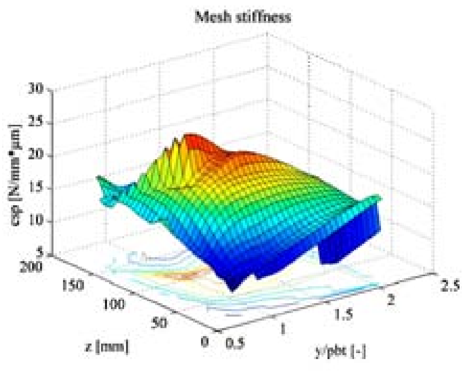


Fig. 3a.
Mesh Stiffness as Function of Teeth Pair Bearing Pattern Distance z [mm] During One Tooth Pair Mesh Period y/p_{bt} [-]

5. Results and Discussion

For modeled gear pair, results for the mesh stiffness c_{sp} ($N/mm\ \mu m$) and normal load distribution Fn (N) are obtained by described procedure and shown on Fig. 3. The results are displayed for the middle gear pair from Fig. 2c during one tooth pair meshing period. On this diagrams the variable y/p_{bt} [-] is ratio of contact point position on line of contact (measured from start contact point) y and transverse base pitch p_{bt}, and the variable z [mm] is distance of contact point form tooth face surface.

The mesh stiffness calculated and shown at Fig. 3a and the real teeth pair bearing pattern lengths B_i (i = 1, 2, 3) taken from the FEM helical gears model are used for the calculation of total mesh stiffness k_z (N/m). The trend of the total mesh stiffness change during the periods with two and three simultaneously meshed gear pairs is shown in Fig. 4. Results shown in Fig. 4 correspond to three tooth pair mesh periods (tooth pair mesh period of three tooth pairs). This diagram has an expected form in accordance with results of other authors obtained for a spur gear pair (Lakota and Stajnko, 2009).

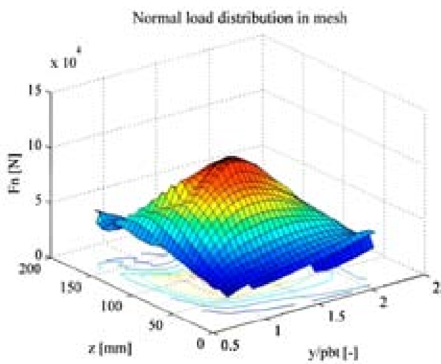


Fig. 3b.
Normal Load as Function of Teeth Pair Bearing Pattern Distance z [mm] During One Tooth Pair Mesh Period y/p_{bt} [-]

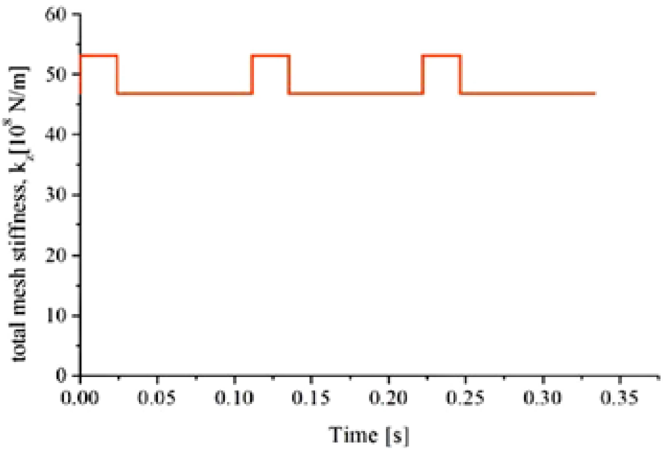
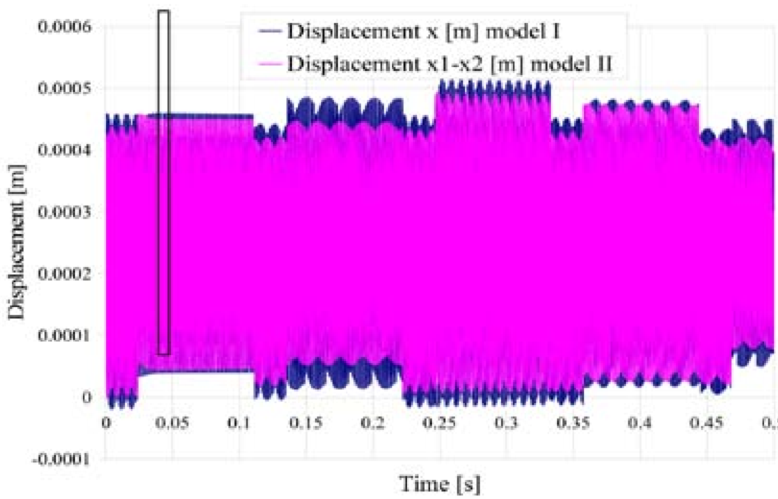
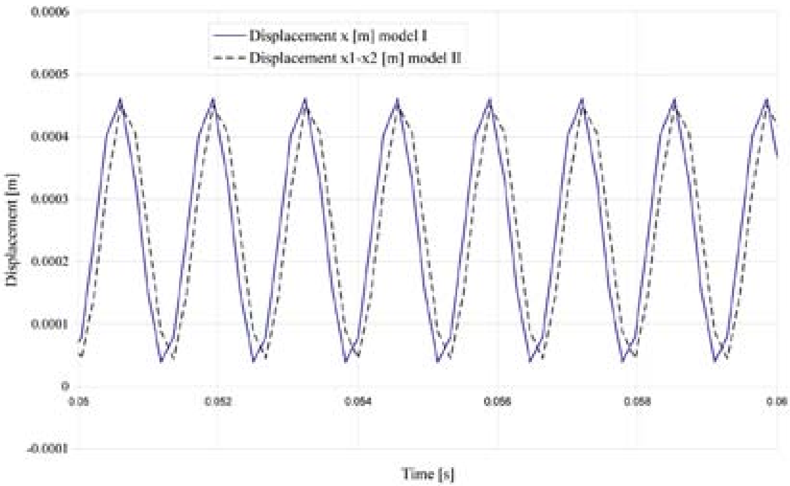


Fig. 4.
Total Mesh Stiffness During Helical Gears Meshing

Obtained results for total mesh stiffness and normal load distribution are then put in nonlinear dynamic analytical model of involutes helical gears motion described with Eq. (1) for model II and Eq. (2) for model I. In that way these analytical models become solvable. The Runge-Kutta numerical iterative methods are used for solving the obtained differential equations. The commercial MATLAB software is used for calculations. The comparative diagrams of results for both of models are shown in Fig. 5a and Fig. 5b.



5a) Displacement x [m] and (x1-x2) [m] as Function of Time During Helical Gears Meshing



5b) A Zoom Detail from Function Shown on Fig. 5a

Fig. 5.
The Results for Nonlinear Dynamics of the Helical Gear Pair

6. Conclusions

This paper shows the new approach for calculation of total mesh stiffness and nonlinear load distribution for helical involute gears. It successfully put together numerical methods for stress and strain calculations and numerical iterative methods for differential equations solving with existing and proven analytical definition of nonlinear dynamics of helical gears. The described mathematical models can be used for the analysis of the gears oscillation parameters, only if all functions are known. In this paper, the stiffness and load distribution are calculated using the developed finite element model. Excellent superposition of diagrams shown in Fig. 5 with results of other authors, (Amezketa et al., 2009; Lakota and Stajnko, 2009; Theodossiades and Natsiavas, 2000), led to the conclusion that developed models and procedures are suitable for future researches. Also, presented research shows that both of analysed nonlinear dynamic analytical models of involute helical gears motion are suitable for future research.

The obtained results for the $x(t)$ function can be used to predict gear noise and increase gears life, and the normal load distribution function can be used for precise helical gears load capacity calculations.

The presented models are suitable for helical gears calculation and investigation when the dimensions and main parameters of gear pairs do not fit to available standard procedures. This is particularly important for gear pairs with high value of transmission ratio, as used in large transport machines.

Acknowledgments

Parts of this research were supported by the Ministry of Sciences and Technology of the Republic of Serbia through Mathematical Institute SANU Belgrade and Institute Kirilo Savic, Belgrade Grant OI 174001 Dynamics of hybrid systems with complex structures, Mechanics of Materials.

References

Amabili, M.; Rivola, A. 1997. Dynamic analysis of spur gear pairs: steady-state response and stability of the SDOF model with time-varying meshing damping, *Mechanical Systems and Signal Processing*, 11(3): 375-390.

Amezketa, M.; Iriarte, X.; Ros, J.; Ma Pintor, J. 2009. Dynamic model of a helical gear pair with backlash and angle-varying mesh stiffness, Multibody Dynamics, *ECCOMAS Thematic Conference Warsaw, Poland.* 17 p.

Andersson, A.; Vedmar, L. 2003. Dynamic model to determine vibrations in involute helical gears, *Journal of Sound and Vibration, 260 (2)*: 195-212.

Atanasovska, I.; Nikolić, V.; Momčilović, D.; Vulović, S. 2007a. Developing of gear FEM model for nonlinear contact analysis. In *Proceedings of 1. International Congress of Serbian Society of Mechanics, Kopaonik*, 695-703.

Atanasovska, I.; Nikolić, V. 2007b. 3D spur gear FEM model for the numerical calculation of face load factor, *The Scientific Journal: Facta Universitatis, Series: Mechanics, Automatic Control and Robotics*, 6(1): 131-143.

Atanasovska, I.; Nikolić, V.; Dimitrijević, D.; Momčilović, D. 2009. Finite element model for stress analysis and nonlinear contact analysis of helical gears, *Scientific Technical Review*, LVIX(1): 61-69.

Dimitrijević, D.; Nikolić, V.; Atanasovska, I. 2007. Dynamic analysis of the stress and strain state of the spur gear pair, *Scientific Technical Review*, LVII(3-4): 20-25.

Faggioni, M.; Samani, F.; Bertacchi, G.; Pellicano, F. 2011. Dynamic optimization of spur gears, *Mechanism and Machine Theory* 46 (4): 544-557.

Hortel, M.; Skuderova, A. 2007. Linear and nonlinear damping in dynamics of gear mesh of the parametric systems with impacts, *Engineering MECHANICS* 14(1/2): 91-104.

Lakota, M.; Stajnko, D. 2009. Application of Joint Time Frequency Analysis for Early Crack Detection on One-stage Gear Drive, *Strojarstvo* 51(6): 565-574.

Parker, R.G.; Vijayakar, S.M.; Imajo T. 2000. Non-linear dynamic response of a spur gear pair: Modeling and experimental comparisons, *Journal of Sound and Vibration* 237(3): 435-455.

Theodossiades, S.; Natsiavas, S. 2000. Non-linear dynamics of gear-pair systems with periodic stiffness and backlash, *Journal of Sound and Vibration* 229(2): 287-310.

DINAMIČKO PONAŠANJE ZUBACA SA VELIKIM PRENOSNIM ODNOSOM

Ivana Atanasovska, Marija Vukšić Popović, Zorica Starčević

Sažetak: U radu je opisano dinamičko ponašanje zupčanika sa kosim zupcima i novi pristup izračunavanju uticajnih promenljivih: krutost spregnutih zubaca, dužina kontaktnih linija i raspodela opterećenja tokom sprezanja. Za proračun deformacija spregnutih zubaca, krutosti spregnutih zubaca i kontaktnih opterećenja korišćena je nelinearna analiza kontakta metodom konačnih elemenata sa iteracijama u toku proračuna. Raspodela normalnog opterećenja koja je dobijena na osnovu ovog postupka korišćena je u proceni nelinearnih dinamičkih analitičkih modela kretanja zupčanika sa kosim zupcima. Opisano istraživanje je od naročite važnosti za zupčaste parove sa velikim prenosnim odnosom, koji se često primenjuju kod velikih transportnih mašina. Predstavljeni modeli i rezultati se mogu koristiti za modeliranje zupčanika sa kosim zupcima kada standardne procedure ne ispunjavaju zahteve.

Ključne reči: zupčanici, transportne mašine, dinamika.