EFFICIENCY MEASUREMENT OF DELIVERY POST OFFICES USING FUZZY DATA ENVELOPMENT ANALYSIS (POSSIBILITY APPROACH)

Ranko R. Nedeljković[1], **Dragana Drenovac**[2]

[1, 2] *University of Belgrade, Faculty of Transport and Traffic Engineering, Vojvode Stepe 305, 11000 Belgrade, Serbia*

Abstract: This article integrates fuzzy set theory in Data Envelopment Analysis (DEA) framework to compute technical efficiency scores when input and output data are imprecise. In conventional DEA inputs and outputs data are precise. However, traffic and transportation take place in an uncertain environment and input and output data might be imprecise. This article proposes a possibility approach for solving DEA models with fuzzy data. For the special case in which fuzzy membership functions of fuzzy data are trapezoidal, the possibility approach transforms fuzzy DEA models into linear programming models. By using a group of five post offices in Serbia, this approach is empirically illustrated.

Keywords: fuzzy data envelopment analysis, possibility theory, possibility measure, efficiency.

1. Introduction

Over the past three decades Data Envelopment Analysis (DEA) has emerged as a useful tool for business entities and organizations to evaluate their activities. DEA evaluates the efficiencies of a set of homogenous decision making units (DMUs) with multiple performance criteria. Data used in evaluating the efficiency usually consists of multiple inputs and multiple outputs which represent possibly conflicting criteria. DEA evaluates the relative efficiency of DMUs by using a ratio of the weighted sum of outputs to the weighted sum of inputs with the weights being variable. DEA identifies the source and the amount of inefficiency in each input relative to each output for a target decision making unit based on peer-group comparisons.

There are some limitations of DEA that have to be considered. Because DEA is a methodology focused on frontiers, small changes in data can change efficient frontiers significantly. Therefore, to successfully apply DEA, we have to have accurate measurement of inputs and outputs. However, in some situations it is difficult to measure inputs and outputs in an accurate way to obtain precise data.

In recent years, fuzzy set theory has been proven to be useful tool for imprecise data in DEA models. The DEA models with fuzzy data, called "fuzzy DEA" models, take the form of fuzzy linear programming models (Lertworasirikul, 2003).

There is no universally accepted approach for solving the fuzzy DEA model. In this paper

---

[1] Corresponding author: r.nedeljkovic@sf.bg.ac.rs

we used possibility approach to solve DEA with imprecise data. We measure efficiency of five Serbian post offices considering two inputs (number of employees and average waiting time) and one output (total number of handled items). Average waiting time is fuzzy input in developed model. Membership functions are of triangular shape. Applying the possibility approach we transform Fuzzy DEA model to a linear programming model and solve by LINDO (software tool for solving linear, integer and quadratic optimization models).

2. Literature Review on DEA and Fuzzy DEA

Many applications of DEA can be found in the literature. The method has been used to evaluate the efficiency of DMUs including banking systems, educational institutions and post offices. DEA has also been applied in various modes of transportation.

Chu et al. (1992) use DEA to measure efficiency of selected bus transit systems in the United States. Boile (2001) extends previous work by considering variable returns to scale and using both a CCR and BCC based approach. She uses DEA to measure efficiency of 23 bus transit systems. Cullinane and Wang (2006) use DEA for measuring the efficiency of 69 container terminals in Europe. Deprins et al. (1995) present a study that evaluates performance of services of 972 Belgian post offices. Doble (1995) measures efficiency in UK post office counters. The study includes 1281 UK post offices. Tongzon (2001) uses DEA to measure efficiency of four Australian and twelve international ports.

Drenovac and Nedeljkovic (2011) applied DEA method on estimating delivery post office efficiency. Knezevic and Lekic (2008) use DEA to measure efficiency of 28 post offices in Prokuplje area and consider functioning

of post office counters and delivery services. Pjevcevic and Vukadinovic (2010) measured efficiency of bulk cargo handling at river port.

Papers that have been published on solving fuzzy DEA problems can be categorized following several distinct approaches. Tolerance approach defines tolerance levels on constraint violations. In the defuzzification approach, fuzzy inputs and fuzzy outputs are first defuzzified into crisp values. The resulting crisp model can be solved by an LP solver. In α-level based approach, the fuzzy DEA model is solved by parametric programming using α-cuts. In fuzzy ranking approach both fuzzy inequalities and fuzzy equalities in the fuzzy CCR model are defined by ranking methods so that the resulting model is a bi-level linear programming model. Possibility approach uses concepts of possibility measures and chance-constrained programming to transform a fuzzy DEA model into a well-defined possibility model. Credibility approach uses the "expected credits" of fuzzy variables to deal with uncertainty in fuzzy objectives and fuzzy constraints. The expected credits of fuzzy variables are derived by using credibility measures.

Lertworasirikul et al. (2003) proposed a possibility approach in which fuzzy constraints are treated as fuzzy events and fuzzy DEA model is transformed into possibility DEA model by using possibility measures on fuzzy events. Mugera (2011) applied fuzzy DEA to compute the technical efficiency scores of 34 DMUs using the α-cut level approach. Nedeljkovic and Drenovac (2008) used fuzzy DEA, credibility approach, to measure efficiency of Serbian post offices.

3. Conventional DEA Model

Data Envelopment Analysis (DEA) is a non-parametric methodology for measuring

efficiency within a group of decision-making units (DMUs) that utilize several inputs to produce a set of outputs. DEA models provide efficiency scores that assess the performance of different DMUs in terms of either the use of several inputs or the production of certain outputs. The input-oriented DEA scores vary in $(0, 1]$, the unity value indicating the technically efficient units.

Suppose that there are *n* decision making units (DMUs), each of which has *m* inputs and r outputs of the same type. All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive. The following notation will be used throughout the paper:

DMU_i is the ith DMU,

DMU_0 is the target DMU,

$x_i \in R^{m \times 1}$ is the column vector of inputs consumed by DMU_i,

$x_0 \in R^{m \times 1}$ is the column vector of inputs consumed by the target DMU,

$X \in R^{m \times n}$ is the matrix of inputs of all DMUs,

$y_i \in R^{r \times 1}$ is the column vector of outputs produced by DMUi,

$y_o \in R^{r \times 1}$ is the column vector of outputs produced by the target DMU,

$Y \in R^{r \times n}$ is the matrix of outputs of all DMUs,

$u \in R^{m \times 1}$ is the column vector of input weights, and

$v \in R^{r \times 1}$ is the column vector of output weights.

In the most frequently used DEA model, CCR model, the multiple inputs and multiple outputs of each DMU are aggregated into a single virtual input and single virtual output, respectively.

For the CCR model, a DMU is inefficient if it is possible to reduce any input without increasing any other inputs and achieve the same levels of output (Cooper et al. 2006). If the efficiency of a DMU is equal to 1, the DMU is weakly efficient (technically efficient).

The CCR model is formulated as the following linear programming model:

$$(CCR) \quad \max_{u,v} \quad v^T y_0$$

$$S.t. \quad u^T x_0 = 1$$

$$-u^T X + v^T Y \leq 0$$

$$u \geq 0$$

$$v \geq 0$$

4. Possibility Measure and Fuzzy Event

Similar to the way that measure theory provides the basis for the probability theory, fuzzy sets could be used as a basis for the possibility theory. Also, fuzzy variable is associated with a possibility distribution in the same manner that a random variable is associated with a probability distribution (Lertworasirikul et al. 2003).

Let $(\Theta_i, P(\Theta_i), \pi_i)$, for each i=1,2,…,n, be a possibility space with Θ_i being the nonempty set of interest, $P(\Theta_i)$ the collection of all subsets of Θ_i, and π_i the possibility measure from $P(\Theta_i)$ to $[0,1]$.

Given a possibility space $(\Theta_i, P(\Theta_i), \pi_i)$ Zadeh (1965) defined a fuzzy variable, ξ, as a real valued function defined over Θ_i with the membership function:

$\mu_{\tilde{\xi}}(s) = \pi(\{\theta_i \in \Theta_i | \xi(\theta_i) = s\})$

$$= \sup_{\theta i \in \Theta i} \{\pi(\{\theta\_i\}) | \xi(\theta\_i) = s\}, \forall s \in R$$

Let $(\Theta, P(\Theta), \pi)$ be a product possibility space such that $\Theta = \Theta_1 \times \Theta_2 \times ... \times \Theta_n$ and from possibility theory

$$\pi(A) = \min_{(i=1,2,...,n)} \{\pi_i(A_i) | A = A_1 \times A_2 \times ... \times A_n, A_i \in P(\Theta_i)\}$$

Let $\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n$ be fuzzy variables and $f_j: R^n \to R$ be real-valued functions, for j=1,…,m. The possibility of the fuzzy event "$f_j(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq 0$, j=1,…,m" is given by

$$\pi(f_j(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq 0, j=1,...,m) =$$

$$\sup_{s_1, s_2, ..., s_n \in R} \{\min_{1 \leq i \leq n} \{\mu_{\tilde{a}_i}(s_i)\} |$$

$$f_j(s_1, s_2, ..., s_n) \leq 0, j=1,...,m\}$$

5. Fuzzy DEA

The CCR model with fuzzy coefficients takes the form of fuzzy linear programming model and is given as

(FCCR)   max $v^T \tilde{y}_0$

     u,v

S.t. $u^T \tilde{x}_0 = 1$

     $-u^T \tilde{X} + v^T \tilde{Y} \leq 0$

     $u \geq 0$

     $v \geq 0$

where $\tilde{x}_0$ is the column vector of fuzzy inputs consumed by the target DMU (DMU_0), $\tilde{X}$ is the matrix of fuzzy inputs of all DMUs, $\tilde{y}_0$ is the column vector of fuzzy outputs produced by the target DMU (DMU_0), $\tilde{Y}$ is the matrix of fuzzy outputs of all DMU_s (Lertworasirikul, 2002).

Chance-Constrained Programming (CCP) deals with uncertainty by specifying the desired levels of confidence with which the constraints hold. Using the concepts of CCP and possibility of fuzzy events, the FCCR model becomes the following possibility CCR (PCCR) model:

(PCCR)   max $\tilde{f}$

     u, v, $\tilde{f}$

S.t. $\pi(v^T \tilde{y}_0 \geq \tilde{f}) \geq \beta$ (1)

     $\pi(u^T \tilde{x}_0 = 1) \geq \alpha_0$ (2)

     $\pi(-u^T \tilde{X} + v^T \tilde{Y} \leq 0) \geq \alpha$ (3)

     $u \geq 0$

     $v \geq 0$

where β and $\alpha_0 \in [0,1]$ are pre-specified acceptable levels of possibility for constraints (1) and (2), respectively, while $\alpha = [\alpha_1, ..., \alpha_n]^T \in [0,1]^n$ is a column vector of pre-specified acceptable levels for the vector of the possibility constraint (3).

The following relations are useful for the solution of the PCCR model:

Let $\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n$ be fuzzy variables with normal and convex membership functions. Let $(\cdot)^L_{a_i}$ and $(\cdot)^U_{a_i}$ denote the lower and upper bounds of the α– level set of $\tilde{a}_i$, i=1,…,n (Fig. 1). Then, for any given possibility levels α_1, α_2 and α_3 with $0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$,

$$\pi(\tilde{a}_1 + ... + \tilde{a}_n \leq b) \geq \alpha_1 \text{ if and only if } (\tilde{a}_1)^L_{\alpha_1} + ... + (\tilde{a}_n)^L_{\alpha_1} \leq b$$

$$\pi(\tilde{a}_1 + ... + \tilde{a}_n \geq b) \geq \alpha_2 \text{ if and only if } (\tilde{a}_1)^U_{\alpha_2} + ... + (\tilde{a}_n)^U_{\alpha_2} \geq b$$

$$\pi(\tilde{a}_1 + ... + \tilde{a}_n = b) \geq \alpha_1 \text{ if and only if } (\tilde{a}_1)^L_{\alpha_3} + ... + (\tilde{a}_n)^L_{\alpha_3} \leq b \text{ and } (\tilde{a}_1)^U_{\alpha_3} + ... + (\tilde{a}_n)^U_{\alpha_3} \geq b$$
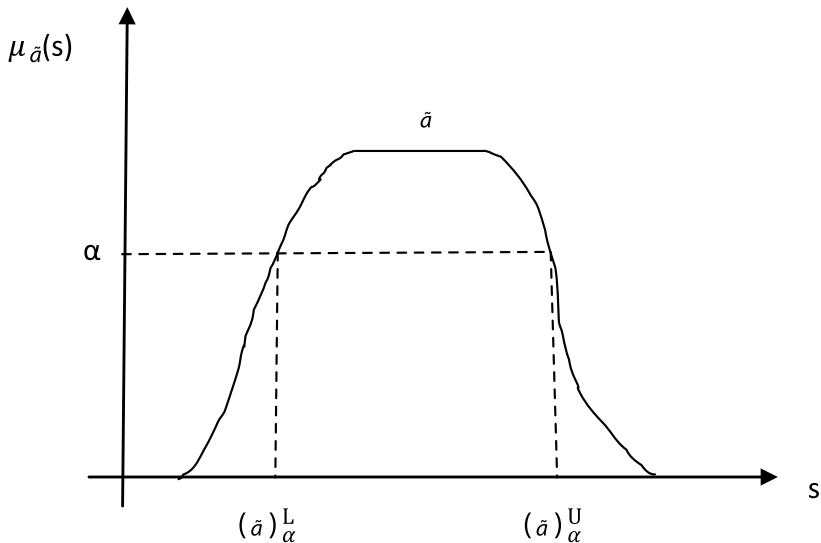
Fig. 1.
The Lower and Upper Bounds of the α-Level of ã

Given that fuzzy inputs and fuzzy outputs of the PCCR model are normal and convex, it follows that the PCCR model can be solved by considering:

$$(PCCR1) \quad \max_{u, v, \tilde{f}} \quad \tilde{f}$$

$$S.t. \quad (v^T\bar{y}_0)^U_\beta \geq \tilde{f}$$

$$(u^T\bar{x}_0)^U_{\alpha_0} \geq 1$$

$$(u^T\bar{x}_0)^L_{\alpha_0} \leq 1$$

$$(-u^T\tilde{X}+v^T\tilde{Y}\leq0)^L_\alpha \leq 0$$

$$u \geq 0$$

$$v \geq 0$$

For the special case, in which fuzzy inputs and fuzzy outputs are trapezoidal fuzzy numbers $\tilde{u}_i=((\tilde{u}_i)^L_0,(\tilde{u}_i)^L_1,(\tilde{u}_i)^U_1,(\tilde{u}_i)^U_0)$ and $\tilde{v}_i=((\tilde{v}_i)^L_0,(\tilde{v}_i)^L_1,(\tilde{v}_i)^U_1,(\tilde{v}_i)^U_0)$ PCCR1 model becomes linear programming model:

$$\max_{u,v,\tilde{f}} \quad \tilde{f}$$

$$S.t. \quad (1-\beta)(v^T\bar{y}_0)^U_0 + \beta(v^T\bar{y}_0)^U_1 \geq \tilde{f}$$

$$(1-\alpha_0)(u^T\bar{x}_0)^U_0 + \alpha_0(u^T\bar{x}_0)^U_1 \geq 1$$

$$(1-\alpha_0)(u^T\bar{x}_0)^L_0 + \alpha_0(u^T\bar{x}_0)^L_1 \leq 1$$

$$(1-\alpha)((-u^T\tilde{X})^L_0 + (v^T\tilde{Y})^L_0) + \alpha((-u^T\tilde{X})^L_1 + (v^T\tilde{Y})^L_1) \leq 0$$

$$u \geq 0$$

$$v \geq 0$$

The objective value $\tilde{f}$ is used to determine if a target DMU is technically efficient in the possibilistic sense at the specified possibility level. Let $\acute{a}$ be the set of β, α_0, α_1,..., α_n. Then, a DMU is $\acute{a}$-possibilistic efficient if its $\tilde{f}$ value at the $\acute{a}$ possibility level is greater than or equal to one; otherwise is possibilistic inefficient.

6. Numerical Example

In this section, Fuzzy DEA is applied to compute the technical efficiency scores of

five post offices (DMUs) in Serbia using the possibility approach. The post offices use two inputs (number of employees and average waiting time) to produce output (total number of handled items). The data were obtained from the Postal Enterprise in Serbia and were also used (Drenovac and Nedeljkovic, 2011) to compute technical efficiencies of the set of Serbian post offices using conventional DEA. In this example, all fuzzy constraints should be satisfied with same possibility level, i.e. $\beta=\alpha_0=\alpha_1=...=\alpha_n$

To illustrate the application of the fuzzy DEA, uncertainty is introduced in the data by representing one input (average waiting time) as symmetric triangular fuzzy number. Membership functions are denoted by (a,b) where a is the center while b is the spread. The data are listed in Table 1.

Table 1

DMUs With One Fuzzy Input

Decision making units	Inputs		Output
	Number of employees	Average waiting time (min) (a,b)	Total number of handled items (10^6)
DMU I	39	(5, 0.5)	2.824
DMU II	81	(9.27, 0.6)	3.238
DMU III	49	(2.63, 0.2)	1.974
DMU IV	73	(5.57, 0.5)	3.975
DMU V	58	(3.45, 0.4)	2.792

For the purpose of efficiency comparison of DMUs, the possibility levels of all constraints should be set at the same level. The fuzzy efficiencies of DMUs were obtained for three

different possibility levels (0, 0.5, 1) and illustrated in Table 2.

Table 2

Results of Efficiency Values at Three Possibility Levels

Decision making units		Possibility levels		
		0	0.5	1
DMU I	Efficiency	1.23	1.12	1
DMU II		1.05	0.77	0.59
DMU III		1.17	0.96	0.93
DMU IV		1.42	1.16	1
DMU V		1.35	1.04	1

From Table 2, DMUs 1, 4 and 5 are possibilistically efficient at all possibility levels, while DMUs 2 and 3 are possibilistically efficient only at possibility level equal to 0. The results show that when the possibility level is increased, the efficiency value is decreased.

7. Conclusion

This paper has utilized a possibility approach as a way to solve fuzzy DEA models. The approach transforms a fuzzy DEA model into a possibility DEA model. A numerical example is given to illustrate the implementation of the approach. Efficiencies of five Serbian post offices are measured, considering two inputs (one is fuzzy) and one output. Membership functions of fuzzy input are of triangular shape. In that case the possibility DEA model has the form of a linear programming model. The model is solved by LINDO solver. A possible topic for further research would be to apply other approaches for solving fuzzy DEA models and to compare the results.

References

Boile, M. 2001. Estimating technical and scale inefficiencies of public transit systems, *Journal of Transportation Engineering* 127(3): 187-193.

Charnes, A.; Cooper, W.; Rhodes, E. 1978. Measuring the efficiency of decision making units, *European Journal of Operational Research* 2: 429-444.

Chu, X.; Fielding, G.J.; Lamar, B. 1992. Measuring transit performance using data envelopment analysis, *Transportation Research Part A: Policy and Practice* 26(3): 223-230.

Cooper, W.; Seiford, L.; Tone, K. 2006. *Introduction to Data Envelopment Analysis and Its Uses*. Springer Science+Business Media Inc. 354 p.

Cullinane, K.; Wang, T. 2006. The efficiency of European container ports: a cross-sectional data envelopment analysis, *International Journal of Logistics: Research and Applications*, 9(1): 19–31.

Deprins, D.; Simar, H.; Tulkens, H. 1995. Measuring Labor Efficiency in Post Offices. In Muchard, P. Pestiean, H. Tuklens (eds.) *The performance of Public Enterprises*, North Holland, 243-267.

Doble, M. 1995. Measuring and improving technical efficiency in UK post office counters using data envelopment analysis, *Annals of Public and Cooperative Economics*, 66(1): 31–64.

Drenovac, D.; Nedeljkovic, R. 2011. Efficiency measurement of delivery post offices using data envelopment analysis, *Tehnika*, 66(2): 275-280, (in Serbian).

Knežević, N.; Lekić, S. 2008. Improving of decision making process in postal system using data envelopment analysis, *Tehnika*, 58(3): 1-12, (in Serbian).

Lertworasirikul, S.; Fang, S.C.; Joines, J., Nuttle, H. 2003. Fuzzy data envelopment analysis (DEA): a possibility approach, *Fuzzy Sets and Systems* 139: 379-394.

Lertworasirikul, S. 2002. *Fuzzy Data Envelopment Analysis*, PhD thesis, Graduate Faculty of North Carolina State University. 105 p.

Lindo. 1992. Lindo Systems Inc. Chicago Il.

Martić, M.; Guberinić, S.; Knežević, N. 2000. Post office efficiency comparing using data envelopment analysis, *PosTel*, Belgrade, 33-43, (in Serbian).

Mugera, A. 2011. Measuring technical efficiency of dairy farms with imprecise data: a fuzzy data envelopment analysis approach. Available from internet: <http://purl.umn.edu/103251>.

Nedeljković, R.; Drenovac, D. 2008. Fuzzy data envelopment analysis application in postal traffic, *PosTel*, Belgrade, 47-56, (in Serbian).

Pjevčević, D.; Vukadinović, K. 2010. Efficiency measurement of bulk cargo handling at river port using data envelopment analysis, *Tehnika*, 65(4): 14-19, (in Serbian).

Tongzon, J. 2001. Efficiency measurement of selected Australian and other international ports using data envelopment analysis, *Transportation Research Part A*, 35: 107-122.

Zadeh, L. A. 1965. Fuzzy sets, *Information and Control*, 8: 338-353.

ODREĐIVANJE EFIKASNOSTI POŠTANSKIH JEDINICA PRIMENOM FAZI ANALIZE OBAVIJANJA PODATAKA (PRISTUP ZASNOVAN NA MERI MOGUĆNOSTI)

Ranko R. Nedeljković, Dragana Drenovac

Sažetak: U ovom radu su za određivanje tehničke efikasnosti u uslovima neizvesnosti korišćene teorija fazi skupova i Analiza obavijanja podataka (DEA). U klasičnoj DEA metodi podaci su precizno definisani. Međutim, saobraćaj i transport se odvijaju u takvom okruženju da podaci mogu biti neprecizni. U ovom radu je za rešavanje Fazi DEA modela predložen pristup zasnovan na meri mogućnosti. U specijalnom slučaju, kada su funkcije pripadnosti fazi podataka trapezoidnog oblika, predloženim pristupom se Fazi DEA model transformiše u model linearnog programiranja. Pomenuti pristup je ilustrovan kroz numerički primer koji obuhvata pet pošta koje pripadaju JP PTT Srbije.

Ključne reči: fazi analiza obavijanja podataka, teorija mogućnosti, mera mogućnosti, efikasnost.