

# RANKING EFFICIENT AND INEFFICIENT DECISION MAKING UNITS IN DATA ENVELOPMENT ANALYSIS

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**Abstract:** Data envelopment analysis is a non-parametric linear programming method capable of the efficiency evaluation of decision making units (e.g. public transport companies). It is very often used in the transport sector for the efficiency assessment of airports, ports, railways and public transport companies. However, the original DEA method does not differentiate the efficient firms and thus, does not create full ranking. To overcome this problem, several methods have been developed with the aim of enlarging the distinguishing power of DEA. The present article aims to review these methods with a special emphasis on the ones elaborated in the last decade.

**Keywords:** data envelopment analysis, full ranking, different techniques.

## 1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric linear programming method used for determining the efficiency of a set of companies as compared to the best practice frontier. It can be employed to analyze organizations. The application of the method in the transport sector is wide-spread, especially in the evaluation of airports, ports, railways and urban transport companies (Markovits-Somogyi, 2011a). The aim of the present article is to review and summarize the most common methods for fully ranking decision making units (DMUs) in data envelopment analysis. As is well known, DEA assigns the efficiency value of one to the DMUs which are strongly or weakly efficient (i.e. in the latter case input or output slacks are present). All the DMUs lying on the efficiency frontier are considered efficient and thus there might be several units with an efficiency value of unity. To be able to distinguish the performance of

these units, numerous ranking methods have been developed since the introduction of the DEA technique. The current review follows the lead of Adler et al. (Adler et al., 2002) who give a very good summary of the ranking techniques available at the given date. Here, however, the focus is shifted towards the developments of the last decade without forgetting to include the main approaches which dominate the field. It must be noted that several of the solutions found in the literature are not anymore distinct measures that can easily be categorized into one or the other group of applications, the approaches frequently overlap. Hence, the aim was to give a clear and concise picture of the models at hand and list them below the heading which is the most revealing as to the content of the method.

## 2. The Basic DEA Model-DEA CCR

In order to be able to review the ranking measures, we have to be familiar with the traditional DEA approach. This is explained in this preliminary

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section. Let us suppose that the number of the DMUs to be evaluated is  $n$ , and each DMU consumes  $m$  different inputs and produces  $s$  different outputs. Thus,  $DMU_j$  consumes  $x_{ij}$  of input  $i$ , and produces  $y_{rj}$  of output  $r$ . The DEA model developed by Charnes et al. (1978), DEA CCR – named after the initials of the authors – can then be mathematically formulated (Eq. (1)) as follows (Cooper et al., 2004).

$$\min \theta - \varepsilon \left( \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \quad (1)$$

subject to:

$$\sum_{j=1}^n x_{ij} \lambda_j + s_i^- = \theta x_{i0} \quad i=1, 2, \dots, m;$$

$$\sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = y_{r0} \quad r=1, 2, \dots, s;$$

$$\lambda_j, s_i^-, s_r^+ \geq 0 \quad \forall i, j, r$$

Here  $\lambda_j$ s are the weights assigned by the linear program,  $\theta$  is the efficiency calculated,  $s_i^-$  and  $s_r^+$  are the input and output slacks, respectively,  $\theta$  is the index of DMU under evaluation, and  $\varepsilon$  is a non-Archimedean element defined to be smaller than any positive real number. So as to contribute to the easier understanding of the nominations, the original model of the program described above is also provided here, Eq. (2):

$$\max h_0(u, v) = \frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{i=1}^m v_i x_{i0}} \quad (2)$$

subject to:

$$\frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1 \quad j = 1, \dots, n; \text{ and } u_r, v_i \geq 0;$$

where  $u, v$  are the weights to be optimized,  $y_{r0}, x_{i0}$  are the observed input/output values of  $DMU_0$  (the DMU to be evaluated).

### 3. Super-Efficiency

Perhaps super-efficiency is the most well known, most widely applied and researched ranking method in DEA. In the field of transport, for instance, Adler and Berechman (2001) evaluate 26 airports of Western Europe, North America, and the Far East with the super-efficiency DEA method, Bazargan and Vasigh (2003) apply super-efficiency to the ranking of 45 US airports, Hirschhausen and Cullmann (2010) treat the problem of outliers with this technique in their study, while Wu and Goh (2010) utilize the method to investigate the efficiency of 21 container ports (Markovits-Somogyi, 2011b). The idea of super-efficiency as developed by Andersen and Petersen (1993), Banker et al. (1989) and Banker and Gifford (1988) is that the best practice frontier is created first without evaluating  $DMU_0$ , and then with its inclusion. Next the extent to which the envelopment frontier becomes extended is investigated. With this procedure,  $DMU_0$  may even be attributed an efficiency value higher than unity. Mathematically the super-efficiency DEA method can be summarized in the following way, Eq. (3):

$$\min \delta_0 = \theta_0^s - \varepsilon \left( \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \quad (3)$$

subject to:

$$\theta_0^s x_{i0} = \sum_{j=1, j \neq 0}^n \lambda_j x_{ij} + s_i^- \quad i = 1, \dots, m;$$

$$y_{r0} = \sum_{j=1, j \neq 0}^n \lambda_j y_{rj} - s_r^+ \quad r = 1, \dots, s;$$

$$\sum_{j=1, j \neq 0}^n \lambda_j = 1$$

$$\lambda_j \geq 0, j \geq 0$$

$$s_i^- \geq 0 \quad i = 1, \dots, m;$$

$$s_r^+ \geq 0 \quad r = 1, \dots, s;$$

$$\theta_0^s \geq 0$$

where  $\delta_0$  is the value to be minimized, and the index  $s$  of  $\theta^s$  refers to super-efficiency.

The problem with super-efficiency DEA is that under certain conditions infeasibility occurs which limits the applicability of the technique. Seiford and Zhu (1999) provided in their article the necessary and sufficient conditions under which the problem becomes infeasible. Being aware of this shortcoming, several authors endeavoured to develop the super-efficiency technique further. Chen (2004) tries to circumvent the problem of infeasibility by utilizing both the input and output oriented DEA model and achieves full ranking by measuring the input saving or output surplus present. Jahanshahloo et al. (2007) rely on the super-efficiency method developed by Hibi and Sueyoshi (1999), and they incorporate a novelty into it: their new ranking system for the appraisal of extreme efficient DMUs is independent of the inefficient

DMUs. Their basic idea is to measure the distance by which the given efficient DMUs move the frontier from the inefficient DMUs. Lotfi et al. (2007) adapt super-efficiency DEA to the case of exogenously fixed inputs as based on the model of Banker and Morey (1986) who have already dealt with the question of exogenously fixed inputs and outputs. Lee et al. (2011) also seek to characterize input savings and output surplus with the scores and strive to increase or decrease the inputs and outputs in a way that the frontier is reached by the DMU which contributed to infeasibility. Chen and Deng (2011) further develop the model of Jahanshahloo et al. (2007) and in their model the efficiency ranking of an efficient unit depends on the efficiency changes of all inefficient units after the given DMU is excluded from the reference set, whereas the performance of an inefficient DMU is characterized by the efficiency values resulting from the exclusion of each efficient unit from the reference set.

#### 4. Cross-Efficiency

In traditional data envelopment analysis the weights evaluating a given DMU are used only to characterize the DMU in question. Sexton et al. (1986) suggested the introduction of the cross-efficiency DEA method which includes a self and a peer evaluation of the DMUs as well. Thus, the individual decision making units are not only assessed by their own weights but the weights of all the other DMUs are also incorporated into the value judgement. In practice this is carried out by a cross-efficiency matrix which is created as follows, Eq. (4):

$$h_{kj} = \frac{\sum_{r=1}^s u_{rk} y_{rj}}{\sum_{i=1}^m v_{ik} x_{ij}} \quad k = 1, \dots, n; j = 1, \dots, n; \quad (4)$$

where  $h_{kj}$  is the score given to unit  $j$  in the DEA run of unit  $k$ , when unit  $j$  is evaluated (Adler et al., 2002), and it is always true that  $0 \leq h_{kj} \leq 1$ . The values in the diagonal of the cross-efficiency matrix,  $h_{kk}$ s are the values returned by the traditional DEA model. To achieve full ranking,  $\bar{h}_k$  the average cross efficiency score of unit  $k$  is introduced, Eq. (5):

$$\bar{h}_k = \frac{\sum_{j=1}^n h_{kj}}{n} \tag{5}$$

This score is more representative of efficiency than the traditional DEA-score, as all the elements of the cross-efficiency matrix are included in it but one has to be aware that at the same time the connection to the multiplier weights is lost (Adler et al., 2002). An interesting extension of the cross-efficiency method is the maverick index  $M_k$  developed by Doyle and Green (1994) which measures the deviation between the DMUs peer scores and the original DEA score in the following way, Eq. (6):

$$M_k = \frac{h_{kk} - e_k}{e_k} \tag{6}$$

where

$$e_k = \frac{1}{n-1} \sum_{j \neq k} h_{kj}$$

The higher this  $M_k$  maverick value, the more the given DMU can be considered a maverick, i.e. its self evaluated score might be high parallel with a low peer evaluation.

### 5. Benchmark Ranking Method

The simplest benchmark ranking method is the approach when it is counted how many times a

given DMU is peer to the other decision making units. Obviously this solution will not necessarily provide full ranking, as the resulting number may be equal in case of different DMUs. A more sophisticated method is developed by Torgersen et al. (1996) which also investigates the extent to which the different DMUs are peers to each other and through this benchmarking procedure full ranking is achieved. The technique consists of two steps: first a traditional, slack based DEA model is solved and  $V$ , the set of efficient DMUs is determined. The DMUs which have slack values of zero belong to this set. Then the following model is applied to all decision making units, Eq. (7):

$$\frac{1}{E_k} = \max \theta_k \tag{7}$$

subject to:

$$-\sum_{j \in V} \lambda_{kj} x_{ij} - s_{ik}^- = -x_{ik} \quad i = 1, \dots, m;$$

$$\sum_{j \in V} \lambda_{kj} y_{rj} - \theta_k y_{rk} - s_r^+ = 0 \quad r = 1, \dots, s;$$

$$\sum_{j \in V} \lambda_{kj} = 1$$

where  $V$  is the set of efficient units.

Finally, the individual reference weights created by the previous model are aggregated and this benchmarking measure ranks the decision making units, Eq. (8):

$$\rho_k^r \equiv \frac{\sum_{j=1}^n \lambda_{kj} (y_j^p - y_j)}{y_r^p - y_r} \tag{8}$$

$$\forall k = 1, \dots, V, r = 1, \dots, s,$$

where

$$y_{rj}^P = \frac{y_{rj}}{E_k} + s_{rj}^+$$

$$u_r \geq \varepsilon \quad r = 1, \dots, s;$$

$$v_i \geq \varepsilon \quad i = 1, \dots, m.$$

where

$$\hat{x}_{ij} = \frac{x_{ij}}{\sum_{k=1}^n x_{ik}} \quad i = 1, \dots, m; j = 1, \dots, n;$$

$$\hat{y}_{rj} = \frac{y_{rj}}{\sum_{k=1}^n y_{rk}} \quad r = 1, \dots, s; j = 1, \dots, n$$

## 6. Common Weights

### 6.1. Applying Minimum Weight Restriction

If developed purposefully, the utilization of common weights can contribute to full ranking. The main goal of Wang et al. (Wang, Luo, Liang, 2009) is to introduce a minimum weight restriction and as a side effect, common weights are also achieved. (Imposed weight restrictions to incorporate value judgement are widely researched within DEA but as these methods originally do not necessarily and purposefully provide a full ranking, they are not explicitly discussed here. The reader is referred to the theory of assurance regions developed by Thompson et al. (1986) and to the cone ratio DEA approach of Charnes et al. (1990) and further developed, for example by Talluri and Yoon (2000).

Wang et al. construct a maximin weight model and use this to reach full ranking, Eqs. (9) and (10):

$$\text{maximize } \varepsilon \quad (9)$$

subject to:

$$\sum_{i=1}^m v_i = 1$$

$$\sum_{r=1}^s u_r \hat{y}_{r0} - \sum_{i=1}^m v_i \hat{x}_{i0} = 0$$

$$\sum_{r=1}^s u_r \hat{y}_{rj} - \sum_{i=1}^m v_i \hat{x}_{ij} \leq 0 \quad j = 1, \dots, n;$$

are the normalized inputs and outputs and  $\varepsilon$  is not the usual, non-Archimedean infinitesimal used in the traditional two-stage DEA model, but a decision variable, not necessarily very small and set to be the maximin input and output weight of DMU<sub>0</sub>.

$$\varepsilon^* = \max_{u_r, v_i} \left\{ \min_r \left( \min_i u_r, \min_i v_i \right) \right\} \quad (10)$$

Thus to achieve full ranking the following steps have to be taken: 1) perform a normal DEA CCR to identify the efficient units 2) run the maximin weight model to find the maximin weight of each DMU 3) set weight restrictions and reassess the efficiencies of the DMUs with the weight restrictions incorporated in the judgement 4) rank the units as based on the new efficiency scores. Apart from utilizing common weights for the achievement of full ranking, the approach of Wang et al. includes further two novelties as compared to the traditional solutions: normalizing the input and output data and using minimum weight restrictions. The maximin theory is also applied by Troutt (1995) who developed a maximin

efficiency ratio model which also creates common weights for evaluation, Eq. (11).

$$\max_{u_r, v_i} \left( \min_k \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \right) \quad (11)$$

subject to:

$$\frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1 \quad j = 1, \dots, n;$$

$$\sum_{r=1}^s u_r = 1$$

$$u_r, v_i \geq 0 \quad r = 1, \dots, s; \quad i = 1, \dots, m.$$

### 6. 2. Statistical Analysis

Common weights are also achieved by different multivariate statistical analyses and these can also lead to full ranking. For instance, canonical correlation, linear discriminant analysis or the discriminant analysis of ratios can be employed. Friedman and Sinuany-Stern (1997) use canonical correlation (CCA) to determine a scaling ratio score, Eq. (12):

$$T_j = \frac{W_j}{Z_j} \quad j = 1, \dots, n; \quad (12)$$

where

$$Z_j = V_1 x_{1j} + V_2 x_{2j} + \dots + V_m x_{mj},$$

$$W_j = U_1 y_{1j} + U_2 y_{2j} + \dots + U_s y_{sj}$$

Here, the task of CCA is to find the vectors  $U'$  ( $U_1, U_2, \dots, U_s$ ) and  $V'$  ( $V_1, V_2, \dots, V_m$ ) which maximize a given coefficient of correlation between the composite input,  $Z$  and the composite output,  $W$ . Sinuany-Stern et al. (1994) utilize linear discriminant analysis and develop the following one-dimensional linear function, Eq. (13):

$$D_j = \sum_{r=1}^s u_r y_{rj} + \sum_{i=1}^m v_i (-x_{ij}) \quad j = 1, \dots, n \quad (13)$$

Value  $D_j$  is used for ranking, where the DMU with the highest  $D_j$  value is the most efficient decision making unit. Both of the above described models are only feasible when the weights are non-negative. Discriminant analysis of ratios combined with a non-linear search optimization algorithm can also be used for ranking but there is no guarantee that the solution found is globally optimal. (Adler et al., 2002) A newly developed approach is that of Wang and Luo (2011) who aim to reach full ranking by the following regression analysis, the authors provide two different models (Eqs. (14) and (15)):

$$\text{minimize } z = \sum_{j=1}^n \left( \theta_j^* - \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \right)^2 \quad (14)$$

subject to:

$$u_r \geq 0 \quad r = 1, \dots, s;$$

$$v_i \geq 0 \quad i = 1, \dots, m.$$

$$\text{minimize } J = \sum_{j=1}^n \left( \sum_{r=1}^s u_r y_{rj} - \theta_j^* \sum_{i=1}^m v_i x_{ij} \right)^2 \quad (15)$$

subject to:

$$\sum_{r=1}^s u_r \left( \sum_{j=1}^n y_{rj} \right) + \sum_{i=1}^m v_i \left( \sum_{j=1}^n x_{ij} \right) = n$$

$$u_r \geq 0 \quad r = 1, \dots, s;$$

$$v_i \geq 0 \quad i = 1, \dots, m.$$

Solving either of the models yields common weights for fully ranking the decision making units.

### 7. Slack Based DEA

Another way of approaching the question of full ranking is taking into account the slacks present in the slack-adjusted DEA model. It was Bardhan et al. (1996) who first created an index as based on the slacks to determine the order of DMUs. The Measure of Inefficiency Dominance (MID) (Eq. (16)) was only utilized to rank the inefficient units. (Some argue that the traditional DEA score is not even appropriate for ranking the inefficient DMUs, as these efficiency values are based on different weights and are thus incomparable.)

$$0 \leq 1 - \frac{\sum_{i=1}^m \frac{s_i^-}{x_{i0}} + \sum_{r=1}^s \frac{s_r^+}{y_{r0}}}{m + s} \leq 1 \tag{16}$$

Tone (2002) suggests a similar approach by creating the index  $\rho$  and utilizing that for the development of a super-efficiency like efficiency measure, where  $\rho$  is the objective function to be minimized. Hence, this method could also be viewed as a variant of the super-efficiency technique (Eq. (17)).

$$\rho = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i^-}{x_{i0}}}{1 + \frac{1}{s} \sum_{r=1}^s \frac{s_r^+}{y_{r0}}} \tag{17}$$

Du et al. (2002) acknowledge the viability of the method by pointing out that the slack-based super-efficiency method is always feasible and they also present an extension to the model.

Chen and Sherman (2004) also use slacks for the development of a non-radial super-efficiency DEA model. With stepwise proportional changes in the inputs and outputs the slacks in the radial method are eliminated and thus full ranking is achieved. It must be noted that this leads to the incorporation of value judgments.

### 8. Multi-Criteria Decision Making Methods

Several efforts have been made in literature to combine or employ in parallel data envelopment analysis and a multi-criteria decision making (MCDM) method. Some authors even argue that DEA itself is a MCDM technique (e.g. (Troutt, 1995)). It shall be noted, however, that MCDM is usually applied prior to decision making or project execution, while DEA is more often utilized for the evaluation of schemes already implemented (Adler et al., 2002). One way of merging MCDM with DEA is the incorporation of preferential information into the model. As mentioned earlier, this can be done for instance by limiting the values of weights (assurance region or cone-ratio models). A different solution may be the selection of preferred input/output targets, or even the creation of hypothetical DMUs. These solutions, however, do not always guarantee full ranking. An example from (Troutt, 1995) can be found in the section ‘Common weights’ as his method can be categorized under both headings. The multi-criteria decision making method most frequently integrated with DEA is analytical hierarchical process (AHP). Sinuany-Stern et al. (2000) develop a combined approach where first a pairwise comparison of the decision making units is carried out using the DEA approach modified to cater for this situation (Eq. (18)). Then, in a second run, the DMUs are also investigated using a cross-efficiency solution (Eq. (19)). The resulting values are

used for the creation of a pairwise comparison matrix serving as the basis of the AHP study.

$$E_{AA} = \max \sum_{r=1}^s u_r Y_{rA} \tag{18}$$

subject to:

$$\sum_{i=1}^m v_i X_{iA} = 1, \sum_{r=1}^s u_r Y_{rA} \leq 1, \sum_{r=1}^s u_r Y_{rB} - \sum_{i=1}^m v_i X_{iB} \leq 0$$

$$u_r \geq 0, r = 1, \dots, s, v_i \geq 0, i = 1, \dots, m.$$

$$E_{BA} = \max \sum_{r=1}^s u_r Y_{rB} \tag{19}$$

subject to:

$$\sum_{i=1}^m v_i X_{iB} = 1, \sum_{r=1}^s u_r Y_{rB} \leq 1, \sum_{r=1}^s u_r Y_{rA} - E_{AA} \sum_{i=1}^m v_i X_{iA} = 0$$

$$u_r \geq 0, r = 1, \dots, s, v_i \geq 0, i = 1, \dots, m.$$

The method developed by Sinuany-Stern et al. (2000) has been applied for instance by Guo et al. (2006) for supply chain evaluation. Royendegh and Erol (2009) also build upon the idea of (Sinuany-Stern et al., 2000) but extend the method to ANP (analytic network process), the more generalized form of AHP. Zhang et al. (2006) combine DEA with AHP for 4PL vendor selection but their approach is different. After the construction of an input-output structure, AHP is utilized for a preliminary data analysis with the help of which the importance of the different criteria is determined. The results of the AHP are then used as preferential information in a modified DEA model. A pairwise comparison matrix is created with

the evolving efficiency values and then AHP is applied again for the evaluation of the matrix.

### 9. Application of Fuzzy Logic

Very interesting and unusual path for ranking is taken by Wen and Li (2009) who aim to utilize fuzzy information in data envelopment analysis and as a side effect full ranking is achieved. The core of their method, the fuzzy DEA model is the following (Eq. (20)):

$$\max_{u,v} \theta = Cr \left\{ \frac{v^T \tilde{y}_0}{u^T \tilde{x}_0} \geq 1 \right\} \tag{20}$$

subject to:

$$Cr \{ v^T \tilde{y}_j \leq u^T \tilde{x}_j \} \geq 1 - \alpha \quad j = 1, \dots, n;$$

$$u, v \geq 0$$

where  $\alpha \geq 0.5$ ,  $\tilde{x}_j$  and  $\tilde{y}_j$  are fuzzy variables characteristic of  $DMU_j$ ,  $u$  and  $v$  are considered vectors and  $Cr$  is the credibility measure. Then  $DMU_0$  is  $\alpha$ -efficient, if  $\theta^* \geq \alpha$ , where  $\theta^*$  is the optimal solution of model (Eq. (20)).

Due to the complexity of the technique, it is difficult to solve it with traditional methods, so a hybrid intelligent algorithm is applied and fuzzy simulations are used together with a genetic algorithm. Another way to combine fuzzy logic and DEA is that of Karsak (1998) and Hougaard (1999), who first run a traditional DEA model and then use fuzzy logic to incorporate expert knowledge into the evaluation. (Adler, 2002)



## 10. Shadow Prices

Alirezaee and Afsharian (2007) also take a different approach to full ranking. Without changing the DEA model, they introduce a new aspect (Eq. (21)):

$$F(x, y) = 0 \tag{21}$$

is seen as the efficient production function, and

$\sum_{r=1}^s u_r y_{rj}$  and  $\sum_{i=1}^m v_i x_{ij}$  is viewed as the total revenue and the total cost for the  $j$ th DMU, respectively. Then (Eq. (22)):

$$\sum_i \frac{\partial F}{\partial x_i} x_i + \sum_j \frac{\partial F}{\partial y_j} y_j = 0 \tag{22}$$

While (Eq. (23)):

$$\left[ \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \right] \leq 0 \tag{23}$$

is called profit restriction for the  $j$ th DMU. Using these formulae, a *balance index* is created which for each DMU is the sum of quantities of profit restrictions of other DMUs.  $DMU_i$  will be ranked higher than  $DMU_j$ , if  $DMU_i$  is efficient, but  $DMU_j$  is not; or in another case, when the efficiency score of both DMUs is the same, but  $DMU_i$ 's balance index is more negative.

## 11. Conclusion

As it can be seen from the detailed descriptions provided in the article, several methods have been developed lately with the aim of providing full ranking within data envelopment analysis. The main solutions include super-efficiency, cross-efficiency, the benchmark ranking method,

application of common weights, the slack based DEA, the multi-criteria decision making methods, the utilization of fuzzy logic and shadow prices. Hence, it can be stated that full ranking is achievable through several ways in DEA and the choice of a specific method would depend on the specific needs of the study in question.

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## RANGIRANJE EFIKASNIH I NEEFIKASNIH JEDINICA ODLUČIVANJA U ANALIZI OBAVIJENOSTI PODATAKA

**Rita Markovits-Somogyi**

**Sažetak:** Analiza obavijenosti podataka je neparametarska metoda linearnog programiranja koja se koristi u vrednovanju efikasnosti jedinice odlučivanja (npr. u kompanijama koje se bave javnim prevozom). Metoda se često koristi u transportnom sektoru u oceni efikasnosti aerodroma, luka, železnica i kompanija koje se bave javnim prevozom. Međutim, originalna DEA metoda ne pravi razliku među efikasnim firmama i samim tim ne nudi potpuno rangiranje. Da bi se prevazišao ovaj problem, razvijeno je nekoliko metoda u cilju poboljšanja DEA metode. U ovom radu su razmatrane pomenute metode sa posebnim osvrtom na one koje su usavršene tokom protekle decenije.

**Ključne reči:** analiza obavijenosti podataka, rangiranje, tehnike.