

AN ANALYSIS OF A TRANSPORT SYSTEM USING NON-AFFINE TRANSFORMATIONS

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Abstract: The authors investigated the transformation possibilities and particularly the usage of non-affine transformations of maps in order to analyse the current status and development possibilities of the Hungarian railway system. In the introduction, the authors overview the theoretical background of map transformations and then present the detailed description of the non-parameter affine transformations and their applicability to the evaluation of railway networks.

Keywords: non-affine transformations, map distortion, analysis of railway infrastructure.

1. Introduction

In many areas of everyday life, it is often necessary to analyse and examine the available geographic information thoroughly. The authors investigated the non-affine transformation of infrastructure maps in order to gain new information. We also examined the basic Hungarian railway infrastructure but the described method can be adapted to other transport modes as well. Nowadays railway reaches its second „golden-age”, at European level more and more funds are available for railway investments in order to increase efficient usage of railroad (Gašparík and Zitrický, 2010). The authors built a method which is able not only to analyse the reduction of travel time as a social benefit of the current system, but is also capable of estimating the social benefits of future investments as well.

In this paper, we deal with the transformations of the map. These complex tasks require the processing of spatial information, which is a huge amount of data. Nowadays it is unquestionable that the tools provided by computer science have to be used. For a better understanding, affine transformations are shown first.

2. Affine Transformations

An affine transformation (*linear transformation*) is used in affine geometry and can be negotiated as part of linear algebra. It is a transformation that preserves angles and changes all distances in the same ratio, called the ratio of magnification. A similarity can also be defined as a transformation that preserves ratios of distances (Bódis, 1999). With an affine transformation the transformed coordinates can

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be derived as a linear function of the original coordinates. Affine transformations are shifting, scale changing, rotating and mirroring. An affine transformation is complex when a combination of the above transformations is used (Goodchild, 1984). In a map analysis only a maximum of 3-dimensional part of the mathematically understandable n-dimensional space is interpreted, Eq. (1).

$$[X' \ Y' \ Z'] = [X \ Y \ Z] \cdot \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} + [P_x \ P_y \ P_z] \quad (1)$$

Where (see Table 1):

- $[X' \ Y' \ Z']$ -transformed coordinates
- $[X \ Y \ Z]$ -original coordinates
- $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$ -matrix of transformation tensor
- $[P_x \ P_y \ P_z]$ -vector of shifting

3. Non-Affine Transformations

The primary application of linear transformations are the cases where the necessary changes and the transformation need to be linear, which means it is necessary that after transformation linear objects remain linear as they were before (Detrekői and Szabó, 1995). Equations describing the properties of such affine-transformations have only maximum the first power of coordinates. If there are non-linear distortions, then higher transformations which can correct these non-linear changes are needed. The transformation of the coordinates is also achieved by means of equations: polynomials are used. A complex polynomial is required to implement the necessary transformation that depends on the rate of distortion, the number and the

Table 1

Basic Affine Transformations

If $A_{11}=A_{12}=A_{13}=A_{21}=A_{22}=A_{23}=A_{31}=A_{32}=A_{33}=0$ then	degenerate transformation
If $A_{12}=A_{13}=A_{21}=A_{23}=A_{31}=A_{32}=0$, and $X' = A_{11} \cdot X$ then $Y' = A_{22} \cdot Y$ then $Z' = A_{33} \cdot Z$ then	change in scale in x axis, or change in scale in y axis, or change in scale in z axis.
If $A_{12}=A_{13}=A_{21}=A_{23}=A_{31}=A_{32}=0$, and $ A_{11} = A_{22} = A_{33} $ then $ A_{11} \neq A_{22} \neq A_{33} $ then	same scaling on every axis, or different scaling on every axis.
If $A_{12}=A_{13}=A_{21}=A_{23}=A_{31}=A_{32}=0$, and $ A_{11} > 1$ then $ A_{22} > 1$ then $ A_{33} > 1$ then	enlargement on x axis, or enlargement on y axis, or enlargement on z axis.
If $A_{12}=A_{13}=A_{21}=A_{23}=A_{31}=A_{32}=0$ and $0 < A_{11} < 1$ then $0 < A_{22} < 1$ then $0 < A_{33} < 1$ then	reduction on x axis, or reduction on y axis, or reduction on z axis.
If $A_{12}=A_{13}=A_{21}=A_{23}=A_{31}=A_{32}=0$ and $A_{11} < 0$ then $A_{22} < 0$ then $A_{33} < 0$ then	mirroring on x axis, or mirroring on y axis, or mirroring on z axis.
If $A_{11} = \cos\varphi$ and $A_{12} = \sin\varphi$ and $A_{21} = -\sin\varphi$ and $A_{22} = \cos\varphi$ then	rotation around the origo with φ

Source: Mélykúti, 2007

relative location of the matching points. These polynomials include not only the first but also the higher power of the original coordinates as well. The exponent gives the highest degree polynomial and that is the degree of transformation. The transformed coordinates are calculated on the basis of the transformation matrix. The matrix used for the transformation is set on the coefficients of the polynomial transformation function (see above). The authors also investigated non-affine transformations of the railway network based on travel time due to its economic importance. Rubber Sheet Method (RSM) was used as based on a “flexible surface” in which the original map points are not uniformly transformed. These transformations are not affine transformations, because the parallel lines are not parallel, even curves can occur.

The “rubber sheet” transformations can be implemented piecemeal as well – they are usually called the patch – so the map can be divided into regions and every part can have its own transformation equation. The equations need to satisfy the continuity condition of parts. Non-linear transformations generally provide greater accuracy. By accuracy we mean that when the transformation is applied to the interface points, they faithfully return the other coordinates of the coordinate-system.

4. Using Non-Affine Transformations for the Analysis of Railway Infrastructure

The authors investigated the distance [km] and travel time [min] on the railway between Hungarian cities. Travel time is a very important quality parameter of passenger transport

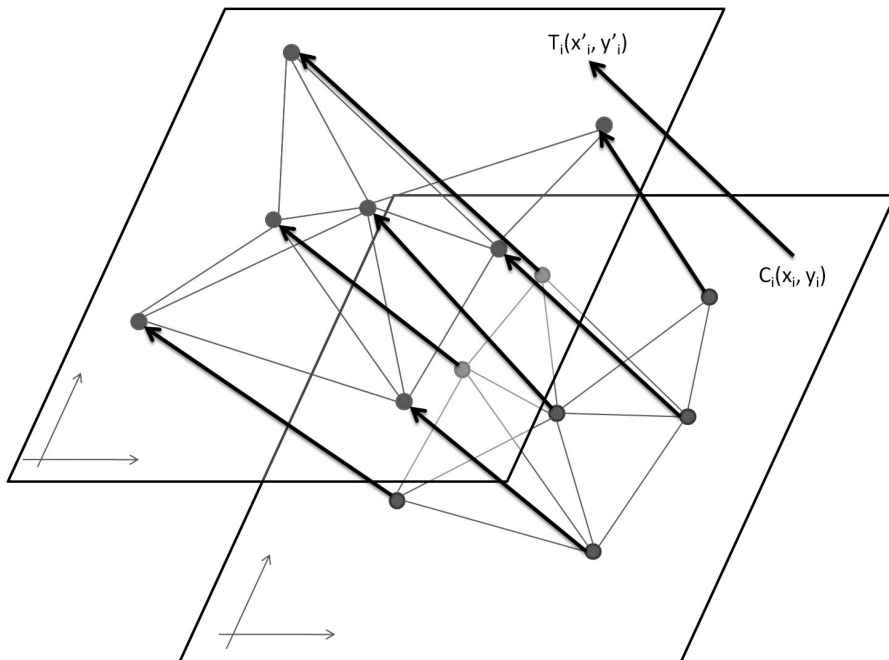


Fig. 1.
 Calculation of Transformation Tensor
 Source: own edition

(Maskeliunaite, et al., 2009). According to the known distances and travel times, two separate and different graphs were built up (Fig. 1). The transformation matrix was calculated on the basis of the matching coordinates. Then the transformation matrix was used to modify the geographical map in order to investigate the railway travel time in Hungary (Fig. 2).

The authors investigated the results gained by using different transformation tensors. The results were statistically analysed. Theoretically, three control points (same points on the geographical and time map) would be enough to have the transformation matrix for calculating the georeference transformation. The authors investigated 34 control points (all county seats and mayor border crossing points for passenger train transport). In the investigation, 34 control points were used, and so a statistical RMS

method was applied in order to compare the transformations. The RMS stands for the *Root Mean Square*, which refers to a distance. The RMS Error value is an error in the distance between the original and the transformed coordinates. When there are 3 control points the RMS Error values are always zero, because an exact mathematical transformation can be calculated. When there are more than 3 control points, then some may show an RMS Error because the georeference calculation is not able to correct a localised deformation of the raster. With the RMS Error values the authors compared the transformation methods. In this investigation the basic (1) equation needs to be extended with the TRMSE (*Total Root Mean Square Error*). The basis of investigation was the minimisation of the TRMSE. The result of the linear transformation Eq. (2) can be seen in Fig. 3:

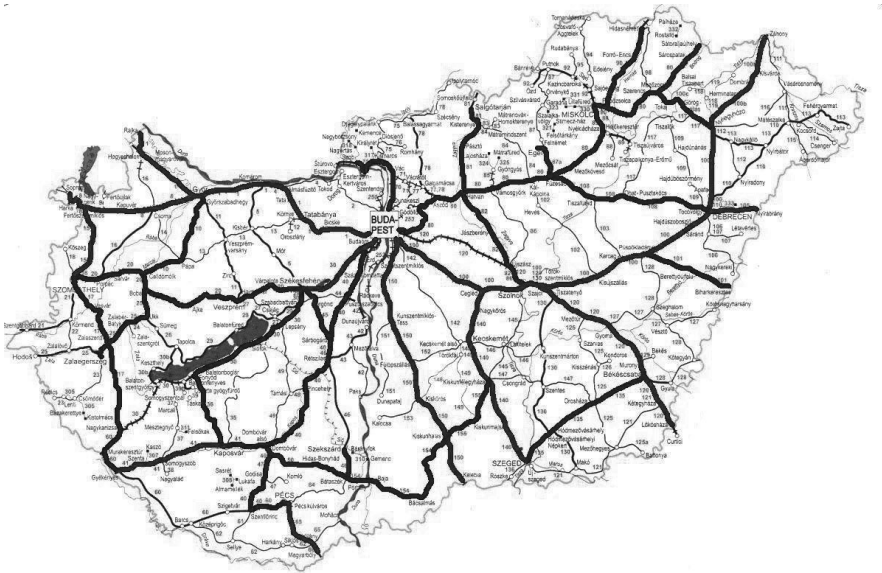


Fig. 2.

Railway Tracks on an Original Unmodified Geographical Map

Source: www.logsped.hu, own edition

$$[X' Y' Z'] = [X Y Z] \cdot \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} + [P_x P_y P_z] + \epsilon_{lin} \quad (2)$$

As it can be seen from Fig. 3 there was only a rotation as a linear transformation to get the control points covered. The statistical results showed that the Total RMS Error was quite high therefore, it needed to be decreased. First the power of the transformation was increased for better approximation. As there were errors and control points were not covered, the authors increased the power of the transformation in order to get a lower error and a better result. The result of the quadratic transformation, Eq. (3) can be seen in Fig. 4:

$$[X' Y' Z'] = [X Y Z]^2 \cdot \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} + [P_x P_y P_z] + \epsilon_{qua} \quad (3)$$

As it can be seen from Fig. 4. linear elements were distorted due to the quadratic transformation to parallelogrammic elements. The statistical results showed the Total RMS Error was smaller in the quadratic case compared to the linear but still significant therefore, it needed to be decreased. Further on, the authors did not increase the power of the transformation for better approximation but chose another way of approximation: the spline transformation which provides the zero Total RMS Error using different distortion tensors for different locations. The result of the spline transformation, Eq. (4) can be seen in Fig. 5:

$$[X' Y' Z'] = \left| [X Y Z] \cdot \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} + [P_x P_y P_z] + \epsilon_{spline} \right|_{x,y,z} \quad (4)$$

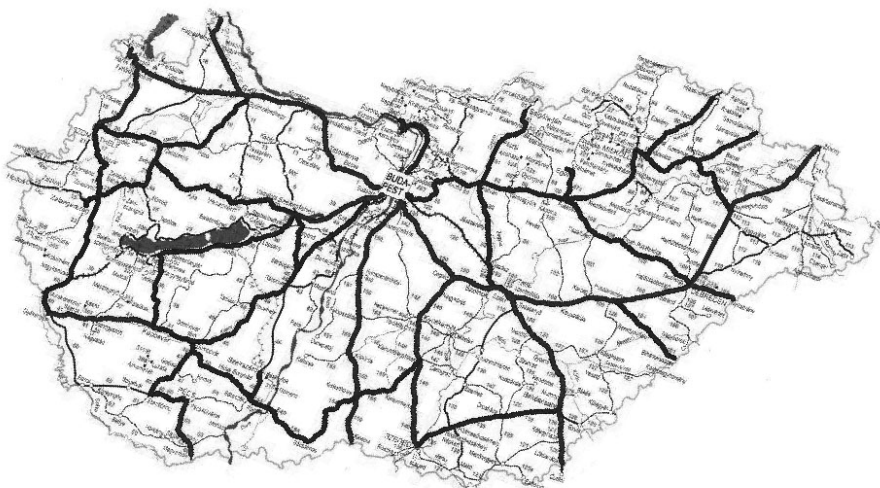


Fig. 3.

Linear Transformation (Total RMS Error= 91.84895) of Railway Tracks by Travel Time
 Source: own edition

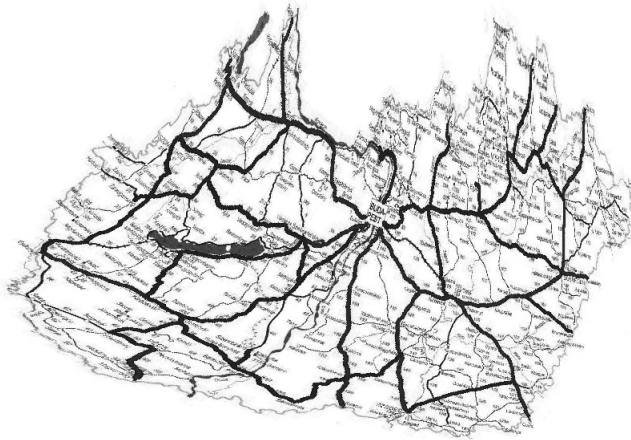


Fig. 4.
Quadratic Transformation (Total RMS Error = 72.93283) of Railway Tracks by Travel Time
Source: own edition

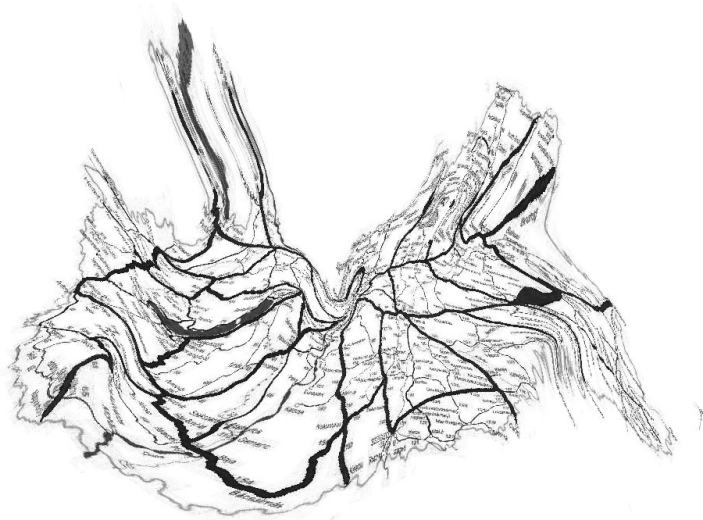


Fig. 5.
Spline Transformation (Total RMS Error = 0.000) of Railway Tracks by Travel Time
Source: own edition

5. Conclusion

In mathematics, the root mean square (RMS), also known as the quadratic mean, is a statistical measure of the magnitude of a varying quantity. It can be calculated for a series of discrete values or for a continuously varying function. The authors investigated the basic transformations and their error coefficient (Table 2):

Table 2

Total Root Mean Square Error Coefficients of Transformations

Type	TRMSE
Linear	91.84895
Quadratic	72.93283
Spline	0.00000

Source: own calculation

The result of the investigation (Fig. 5) clearly shows the centralised position of the capital Budapest and the travel time distortion. As a result, it can be stated that map distortion is fully functioning as a tool of a railway infrastructure investigation. Besides the one that was introduced, other methods are available for analysing (Jarašuniene, 2009) the effect of further investments on travel time.

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ANALIZA TRANSPORTNOG SISTEMA PRIMENOM NE-AFINIH TRANSFORMACIJA

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Sažetak: U radu su ispitane mogućnosti transformacije i naročito primena ne-afinih transformacija mapa u analizi trenutnog položaja i mogućnosti razvoja Mađarskog železničkog sistema. U uvodu je dat teorijski pregled transformacija mapa, a zatim je predstavljen detaljan opis neparаметarskih afinih transformacija i njihova primena u procenjivanju železničkih mreža.

Ključne reči: ne-afine transformacije, distorzija mape, analiza železničke infrastrukture.