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THE INFLUENCE OF CIRCULATING AND ENTERING FLOWS ON CRITICAL GAPS VALUE IN ROUNDABOUTS

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Abstract: The gap acceptance method is one of the most widely used to analyze the capacity of roundabouts. The critical gap has a prominent role in this approach. In every country, driver behavior and local rules are examined and implemented in the local standard for capacity estimation. Hence, a reliable technique for assessing critical gaps can be of great importance. This paper presents an experimental investigation and analysis on whether it is possible to find the correlation between video-based gap acceptance parameters and some traffic parameters in Hungary. Thirteen single-lane roundabouts with different traffic flow rates were recorded for hours in various locations in and around Budapest to estimate the gap acceptance parameter (critical gap) and relate it to circulating and entry flow or the combination of both. Using only linear regression analysis, as a first step, no strong correlation was found between the critical gap and circulating flow and a lower correlation between the critical gap and entry flow. After implementation of a gradient boosted decision tree function, a stronger correlation was found between the critical gap and circulating flow, and an improved correlation between the critical gap and entry flow. The implemented correlation model shows a promising correlation between the critical gap and traffic parameters (circulating and entry flow). Our results indicate that the critical gap has a higher correlation with the circulating flow, and with the increase of the circulating flow, the critical gap value tends to decrease.

Keywords: gap acceptance, roundabout, critical gap, circulating flow, entry flow, linear regression.

1. Introduction

Roundabouts are intersections with a standard circular shape and are distinguished by yield on entry and movement around a central island (counterclockwise in Hungary and clockwise in the UK). Roundabouts have been used successfully all over the world (TRB, 2010; Thai Van and Balmefrezol, 2000; Werner and Stuwe, 1991; TRB National Research Council, 1998).

Roundabouts are considered as one of the main elements in traffic networks. Experts and transportation authorities have developed several design standards in the last decades. The capacity, performance, and safety are among the main elements of the roundabout's design (TRB, 2010; Brilon et al., 1997; Guichet, 2005; Stuwe, 1991).

The capacity of a roundabout is usually determined by geometric parameters (central

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island diameter, width of the circulatory path, entry width, splitter island width, etc.) (Kimber, 1980) or driver behavior parameters.

Driver behavior parameters are based on gap acceptance theory. In which, experience has shown that driver behavior patterns are formed by various traffic rules, cultural, and traditions perspective. These driver behavior patterns have a significant impact on capacity, performance, and safety not only on roundabouts also, on different elements of road networks (Brilon, 2011; Kang *et al.,* 2019; Tamas and Torok, 2009).

In term of roundabouts, a driver intending to merge must first evaluate the gaps that become available to determine which gap (if any) is large enough in his or her opinion to accept (see Fig. 1). Accepting that gap implies that the driver believes that he or she is able to complete the merging maneuver and safely join the mainstream within the gap's length. This phenomenon is known as gap acceptance (Garber *et al.,* 2019).

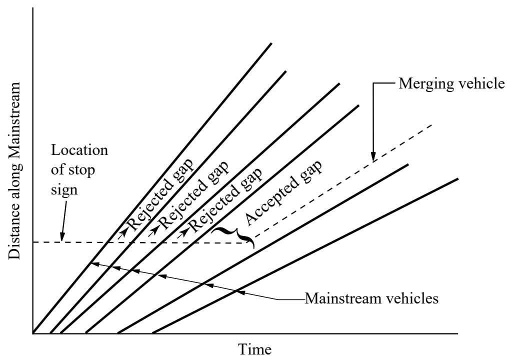

Fig. 1.
Time-Space Diagrams for Vehicles in The Vicinity of a Stop Sign
Source: (Garber et al., 2019)

In the gap acceptance theory, driver behavior is the main role of the theory. Gap acceptance theory is widely used by experts to describe the traffic behavior at roundabouts based on driver behavior. The two main components of gap acceptance theory are critical gap (t_c) and follow-up headway (t_f). Critical gap (t_c) is the smallest period of time an entering Vehicle will accept to merge in the mainstream (Fig. 2). Follow-up headway (t_f) is the amount of time it takes for two consecutive queued Vehicles to enter the same gap in the mainstream (Yap *et al.,* 2013) as illustrated in (Fig. 2).

Troutbeck (2014) definition of the critical gap is the smallest gap that a driver is assumed to accept. Thus, the critical gap would be determined by the traffic characteristics and volume of minor-stream flow in addition to mainstream flow. Moreover, it is commonly considered in standard gap acceptance theories. The mainstream Vehicles have priority over minor-stream Vehicles. Minor-stream Vehicles are expected to accept any gap greater than critical gap and reject any gap smaller than the critical gap, as shown in (Fig. 1).

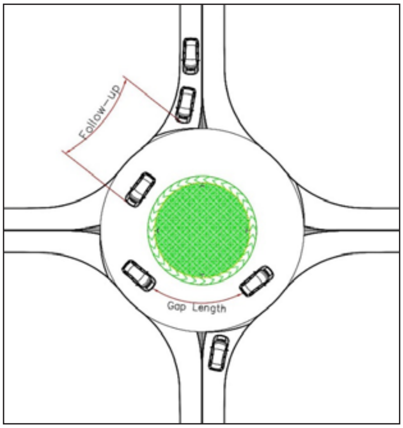

Fig. 2.
Gap Acceptance Parameters (Gap Length, Follow-Up)

Since the critical gap cannot be estimated at the site, there are several methods in the literature used for estimation, such as Raff's method, Ashworth method, Wu model, and maximum likelihood method (Raff MS, 1950; Ashworth, 1970; (Wu, 2012; Tian *et al.,* 1999).

Capacity calculation mainly depends on critical gap and follow-up time values. These values are depending on driver behavior at a certain location or country. Hence, the capacity calculation formulas should not be transferred from one country to another without first finding the main parameters of gap acceptance to examine the driver's behavior at the country of interest (Werner and Stuwe, 1991). As a result, many previous studies have been conducted to estimate the critical gap under different traffic conditions in several countries. For example, the critical gap for single-lane roundabouts in two different cities in Japan (Tokyo and Nagano) were ranged from 3.1-6.6 seconds and 3.0-3.8 seconds, respectively (Manage *et al.,* 2003; Macioszek, 2020). While in India, under Indian traffic conditions, the critical gap was found to be 1.6 seconds, and that might be due to the high number of predominate motorized two-wheelers, which is more than 50% of the Vehicles at both sites of the study (Mathew *et al.,* 2017). In Germany, the critical gap value was estimated to be between 4.0-4.2 seconds (Werner and Stuwe, 1991). In the USA it has been estimated to be between 4.2-5.9 seconds (NCHRP, 2007; TRB, 2007). In Italy the critical gap value for single-lane roundabouts according to Gazzari was between 3.5-4.1 seconds (Gazzarri *et al.,* 2013). In Qatar, the estimated critical gap value for single lane roundabouts was 2.24 seconds (Khaled and Hamad, 2020). And in Poland, the estimated critical gap presented in (Macioszek, 2020) was between 3.16-6.05 seconds.

Data collection is an important factor of any traffic engineering research, and the quality of the data is critical to the project's success. It is essential to properly identify study locations to collect valuable data.

After estimating the critical gap using Raff's method and finding the origin-destination matrix, the aim of this research is to find the effect of entry flow and circulating flow on critical gap value. Raff's method was preferred due to its simplicity, solid results, and straightforward approach (Al Hasanat and Schuchmann, 2022).

The limitation of this article is that the available data set for this research was only for single-lane roundabouts against one single circulating lane.

2. Methodology

In this study, thirteen single-lane roundabouts consisting of forty-one entries were selected in urban and rural areas in Hungary (Fig. 3). The selection was based on the traffic volume variation to perfectly fit the goal of this study. The selected roundabouts have different geometric features (inscribed diameter, entry width, etc.) (Table 1).

The collected data of the selected roundabouts was attained using a drone and a video camera. The video camera is placed at one of the roundabout's entries on a 4 m long pole. The camera is placed where a full view is achieved during peak hours (Fig. 4). The camera specification is described in Table 2. No traffic accident occurred during the recorded period.

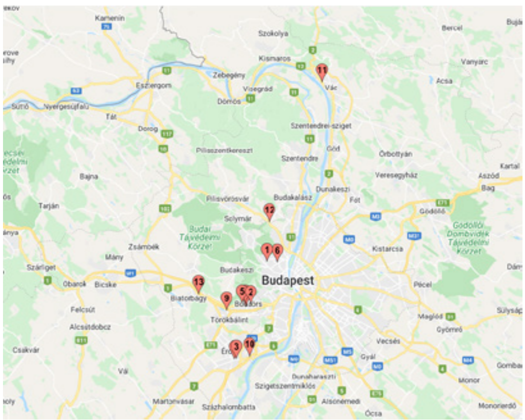

Fig. 3.
The Locations of the Selected Roundabouts in Hungary

Table 1
The Selected Roundabouts Characteristics, Location, Date of Recording, Length of Recording, and Number of Observations

Locations	Coordinates	Inscribed Diameter, m	Length of Recordings	Date of Recording	No. of Observations
Pasaréti tér	47.52391, 18.99338	24	4h:15m	19/10/2020-2/11/2020	576
Szabadság út-Baross utca	47.4608, 18.95833	24	2h:07m	22/03/2021	183
Érd-alsó	47.37861, 18.92618	26.5	2h	17/11/2020	138
Érd felső	47.37819, 18.92191	26.5	1h:30m	9/3/2021	565
Budaörs Gimnázium	47.46066, 18.939	24	2h:30m	23/03/2021	87
Pusztaszeri körönd	47.52305, 19.01688	22	4h	9/10/2020-16/03/2021	385
Budaörs Townhall	47.46116, 18.94561	23	2h:16m	22/03/2021	259
Budaörs Ifjúság	47.45677, 18.94791	26.5	2h:20m	23/03/2021	487
Törökbálint	47.45138, 18.90483	46.5	2h:40m	16/03/2021	321
Nagytétény	47.38302, 18.95497	68	2h	27/01/2021	535
Vac	47.79138, 19.11654	28	1h:30m	14/10/2020	186
Solymárvölgy	47.58343, 18.99947	55	1h:40m	9/3/2020	243
Biatorbágy	47.47516, 18.84341	60	6h:30m	17/03/2020-18/03/2020	340
Σ			35.3 hours		4305

Table 2
Camera Specification

Specifications	
Manufacture	SJCAM
model	SJ4000 WIFI ACTION CAMERA
Sensor	12.0MP CMOS sensor
Lens	170 Degree HD wide-angle Lens
Resolution of videos recorded	1080P (1920*1080) 30FPS 720P (1280*720) 60FPS 720P (1280*720) 30FPS WVGA (640*480) 60FPS

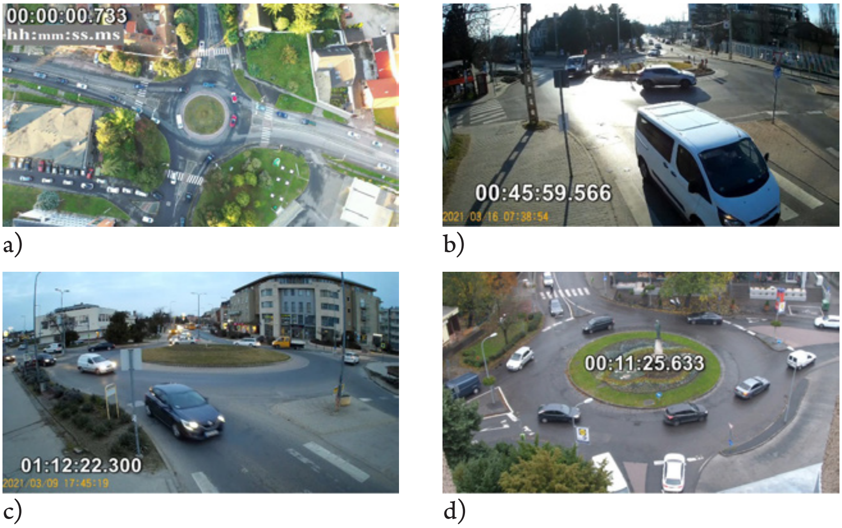

Fig. 4.
A Sample of the Recorded Videos with Timestamp from Different Locations: a): Vac, b): Pusztaszeri Köröndt, c): Érd Felső, and d): Pasaréti Tér
Source: Author

This research investigates the influence of circulating and entry traffic flow of each roundabout's entry on critical gap value by following the illustrated research workflow in (Fig. 5).

This research is divided into four main parts as follows:

- Critical gap analysis of each roundabout entry by applying Raff's method;
- Extracting traffic flow volumes from the recorded videos. For each roundabout entry, both circulating flow and entry flow were analyzed separately;
- Conducting Statistical analysis of the collected data; and
- Results discussion and conclusion.

3. Video Analysis

3.1. Critical Gap Analysis

Gaps data were extracted manually from the recorded videos. A timestamp is added to each video in hh:mm: ss.ms format as illustrated in (Fig. 4). The time gap between the leading vehicle rear bumper (t_n) and front bumper of the following vehicle (t_{n+1}) at the mainstream was recorded in a spreadsheet. Then the $t_{a\,or\,r}=t_{n+1} - t_n$ is calculated using Excel spreadsheet.

Where:
t_n: The time when the leading vehicle passing the conflict point in ss.ms
t_{n+1}: The time when the following vehicle

passes the same conflict point in ss.ms
$t_{a\,or\,r}$: The accepted or rejected gap in ss.ms

Each gap is assigned as an accepted gap (a) or rejected gap (r).

- If the driver accepts the proposed gap and safely joins the mainstream - is marked (a);
- If the driver refuses to join the mainstream and rejects the gap - is marked (r).

After data extraction, the critical gap values of the drivers for the forty-one entries were estimated using Raff's graphical method.

Raff's method is the earliest estimating method of the critical gap based on the research done in (Raff MS, 1950). Raff's method contains the empirical distribution of the functions of accepted gaps $F_a(t)$ and rejected gaps $F_r(t)$. Based on the definition of Raff, the critical gap (*tc*) is the function of time (*t*), and the estimated value of the critical gap is where the functions 1-Fr(t) and Fa(t) intersect see (Fig. 6) (Troutbeck, 2016).

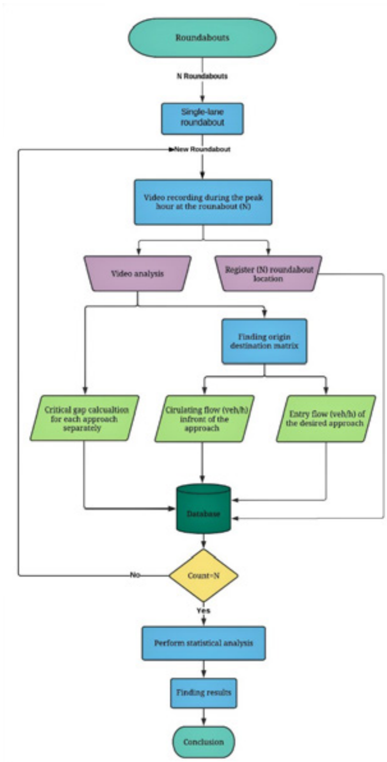

Fig. 5.
Proposed Workflow of the Research Methodology

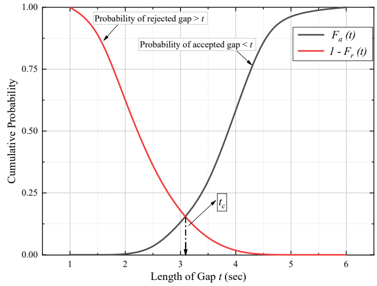

Fig. 6.
Raff's Graphical Method for Estimating the Critical Gap

Raff's method can be described mathematically in the following form (1) (Brilon *et al.*, 1999):

$$1 - F_r(t) = F_a(t) \quad (1)$$

Where:
F_a (t): is the probability of accepted gaps as a function of time (t); and
F_r (t): is the probability of rejected gaps as a function of time (t).

Some examples of the critical gaps estimation at four different locations using Raff's method are illustrated in (Fig. 7). This process of critical gap estimation was performed for all roundabouts entries of this study.

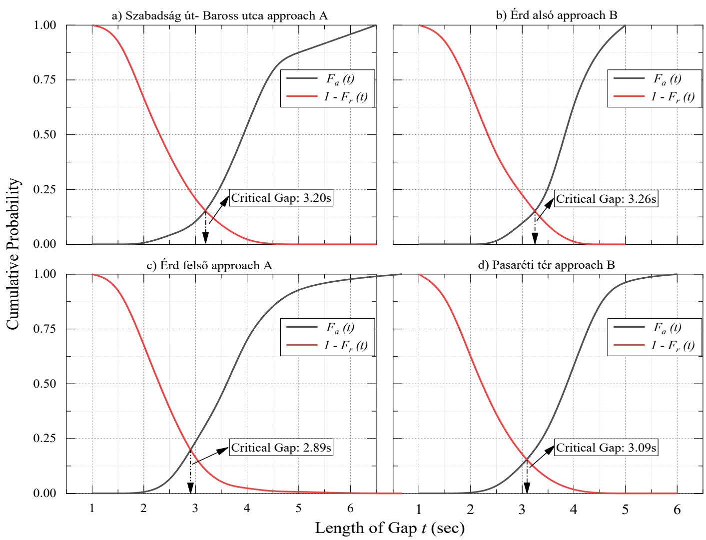

Fig. 7.
Critical Gap Estimation of Four Different Entries at Four Different Locations

The selected roundabouts characteristics, location, date of recording, length of recording, and number of observations are in Table 1.

3.2. Origin Destination Matrix O/D

Obtaining the origin-destination matrix was done by analyzing the recorded videos to acquire the traffic flow data throughout the roundabouts. The traffic counting can either be manual or automated (using Pneumatic tube or Inductive loops), and it is measured every fifteen minutes. In this research, the traffic count was carried out manually by the author using the previously recorded videos to extract the circulating flow, as well as entry flow in passenger car unit (PCU). The passenger car unit (PCU) values for different Vehicle type used in the traffic volume counting are summarized in Table 3.

Table 3
PCU Values of Different Vehicle Class

Vehicle class	PCU
Car, pick up	1.0
Small Truck, Bus	2.0
Cycle	0.5
Motorcycle	0.75
Articulated Bus	3.0

Every recorded video's hour was analyzed in four periods of fifteen minutes for every roundabout's entry. Then the data is recorded in a spreadsheet. Subsequently, the origin-destination matrix $M_{O/D}$ is calculated. Therefore, the entry flow (Q_e) can be easily found, and as it is known that the roundabout is preserved system, the circulating flow (Qc) can be calculated using $M_{O/D.}$ For example, Pasaréti tér traffic Vehicle composition is found (Fig. 8), and roundabout traffic flow Matrix is evaluated and tabulated in Table 4.

Then entry flow (Qe) and circulating flow (Qc) for each entry can be calculated based on the traffic volumes.

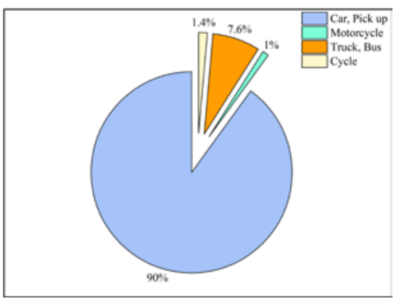

Fig. 8.
Traffic Vehicles Composition at Pasaréti Tér

Table 4
Pasaréti tér O/D Matrix

Pasaréti tér Traffic Volumes in PCU/h					
O/D	A	B	C	D	Total Entry Flow
A	0	167	126	122	415
B	166	0	37	464	667
C	237	190	0	8	435
D	111	448	13	0	572

$$M_{O/D} = \begin{bmatrix} Q_{AA} & Q_{AB} & Q_{AC} & Q_{AD} \\ Q_{BA} & Q_{BB} & Q_{BC} & Q_{BD} \\ Q_{CA} & Q_{CB} & Q_{CC} & Q_{CD} \\ Q_{DA} & Q_{DB} & Q_{DC} & Q_{DD} \end{bmatrix} = \begin{bmatrix} Q_{eA} \\ Q_{eB} \\ Q_{eC} \\ Q_{eD} \end{bmatrix}$$

$$M_{O/D} = \begin{bmatrix} 0 & 167 & 126 & 122 \\ 166 & 0 & 37 & 464 \\ 237 & 190 & 0 & 8 \\ 111 & 448 & 13 & 0 \end{bmatrix} = \begin{bmatrix} 415 \\ 667 \\ 435 \\ 572 \end{bmatrix}$$

$Q_{cA} = Q_{DB} + Q_{DC} + Q_{CB} = 651$ pcu/h
$Q_{cB} = Q_{AC} + Q_{AD} + Q_{DC} = 261$ pcu/h
$Q_{cC} = Q_{BD} + Q_{BA} + Q_{AD} = 752$ pcu/h
$Q_{cD} = Q_{CA} + Q_{CB} + Q_{BA} = 593$ pcu/h

Note that the previously mentioned process of finding the origin-destination matrix was carried out for all the selected roundabouts in this research.

3.3. Statistical Analysis

After finding all the necessary data for the research, a statistical analysis was carried out. There are many statistical methods used to analyze and process the obtained data from recorded videos. Here, since there is only one dependent variable which is the critical gap, along with two independent variables (circulating and entry flow), a multiple regression analysis was implemented using Python, Jupyter notebook.

The purpose of using this statistical method is to find the correlation between the critical gap and both circulating and entry flow, as well as their impact on the critical gap value under different traffic conditions.

4. Analysis and Results

4.1. Data Preparation

Roundabout (location, number, and entry letter) beside critical gap, circulating, and entry flow for each roundabout's entry is formulated in spreadsheet Table 5.

The calculated critical gap values corresponding to each roundabout entry are plotted in (Fig. 9). The mean (μ) and standard deviation SD (σ) values of the data show that 68% of the data fall within ±1 SD (σ) from the mean(μ).

Table 5
Characteristics and Traffic Flows of the Investigated Roundabouts

Roundabout			Critical Gap (sec.)	Circulating Flow (PCU/h)	Entry Flow (PCU/h)
Name	No.	Entry			
Pasaréti tér	1	A	3	651	415
		B	3.09	261	667
		C	2.63	752	435
		D	2.69	593	572
Szabadság út- Baross utca	2	A	3.46	153	534
		B	3.2	403	336
		C	3.37	164	439
Érd-alsó	3	A	3.43	212	416
		B	3.26	217	574
Érd felső	4	A	2.89	1056	162
		B	2.93	542	594
		C	3.07	310	792
		D	3.09	610	598
Budaros Gimnazium	5	A	3.03	361	150
Pusztaszeri körönd	6	A	3.073	517.5	682.5
		B	2.41	884.88	442.5
		C	3.16	637.2	592.5
		D	2.5	756	585
Budaros Town Hall	7	A	3.16	331.2	315
		B	3.23	248.4	510
		D	3	262.8	570
Budaros Ifjúság	8	B	2.91	594	225
		C	3.18	468	300
		D	2.88	525.6	352.5
Torokbalint	9	A	3.2	172.8	720
		B	3.23	1008	180
		C	3.12	212.4	772.5
Nagytétény	10	A	3.15	288	547.5
		B	3	504	90
		C	2.65	612	161.25
		D	3.07	225	438.75
		E	3.14	360	202.5
Vac	11	A	3	810	442.5
		B	2.82	742.5	637.5
		C	2.8	765	397.5
		D	3	479.88	825
Solymárvölgy	12	A	2.9	612	787.5
		B	2.86	576	937.5
Biatorbágy	13	A	3	450	720
		B	3.14	792	712.5
		D	3.2	108	105

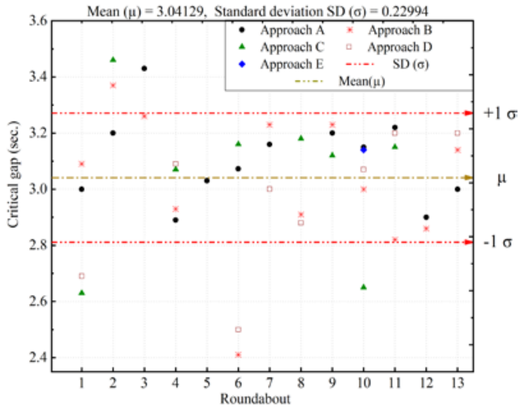

Fig. 9.
Critical Gap for all Roundabouts Entries, with Mean and Standard Deviation

Therefore, the summary of the main statistical parameters of the data was calculated using Python as shown in Table 6.

Table 6
Statistical Characteristics of Collected Data

Main features of the data (Number of data entries, mean, standard deviation, and quantiles)			
	Critical Gap (sec.)	**Entry Flow (PCU/h).**	**Circulating Flow (PCU/h).**
count	41	41	41
mean	3.04129	488.573	493.37
std	0.22994	220.397	245.544
min	2.41	90	108
25%	2.91	336	262.8
50%	3.09	510	504
75%	3.2	667	637.2
max	3.46	937.5	1056

4.2. Data Analysis and Results

The evaluation of the extracted data was carried out using different packages in Python, such as Pandas, Seaborn, and sklearn. Hence, the correlation coefficient R for both independent parameters (circulating and entry flow) to the dependent parameter (critical gap) was visualized with the help of seaborn's heatmap (Fig. 10).

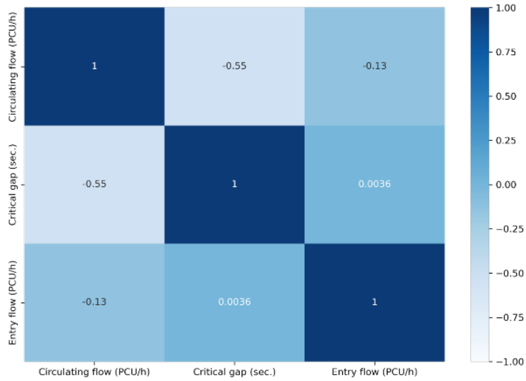

Fig. 10.
Correlation Heatmap of the Investigated Parameters

The results of the correlation of parameters show that the circulating flow has a negative correlation of -0.55, which means with the increase of the circulating flow, the estimated critical gap value tends to decrease. On the other hand, entry flow has a very low correlation R coefficient of 0.0036. More visual representations of the regression line for both cases are plotted in scatter plot (Fig. 11).

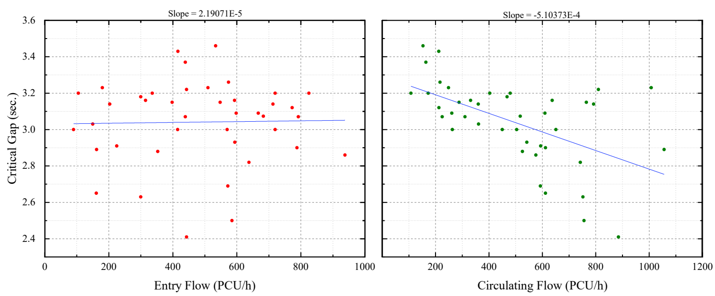

Fig. 11.
Scatter Plot of all the Circulating and Entry Flow Versus Critical Gap

The abovementioned model was computed using the linear regression function of sklearn package, and the value of the coefficient of determination R^2 of the model was 0.302 (~30.2%) (Fig. 12).

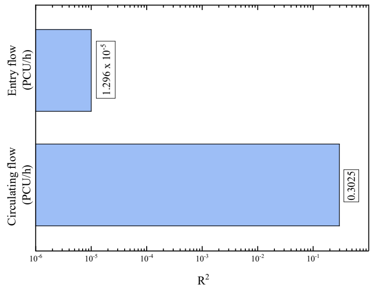

Fig. 12.
Coefficient of Determination R^2 of the Model

Since the results of the model linear regression are not too high, other steps were executed to improve the accuracy of the results using an artificial intelligence predictor such as gradient boosted decision tree in Python. Firstly, the mean squared error of the model was calculated with a value of 0.004759, then after implementing the gradient boosted decision tree function, the value of the returned coefficient of determination (R^2) became 0.9077 (~ 90.8%) (Fig. 13). The implementation of a gradient boosted decision tree function provides a clearer understanding of the data when the mean squared error is eliminated.

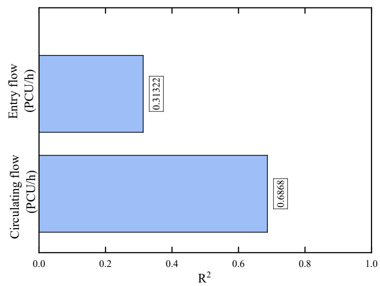

Fig. 13.
Coefficient of Determination R2 of the Model after Gradient Boosting Tree

Furthermore, the partial dependence plot gives a clearer idea of the dependence of circulating flow and entry flow on critical gap (Fig. 14). It is found that with the increase of the circulating flow, the value of the critical gap decreases significantly. On the other hand, the value of the critical gap is not significantly affected when the entry flow increases. These results of the statistical analysis are supported with the results obtained by (Rahmi and Troutbeck, 1991) and (Troutbeck and Soichiro, 1999) found that minor-stream drivers tend to accept shorter critical gaps when the mainstream flow increases.

Moreover, the partial dependence heatmap (Fig. 15) was generated and divided into seven zones. The skewed lines represent different margins of zones from A to G.

Each zone has an upper and lower critical gap value. For traffic practitioners, the generated partial dependence heatmap makes it easier for them to find the critical gap value by simply finding the circulating flow and entry flow volumes. And it could be an alternative way to find the critical gap value quicker than the other time-consuming methods.

For example, assuming traffic flow is such that: the circulating flow is equal to 500 pcu/hour and the entry flow equal to 500 pcu/hour then the critical gap value lies in zone C between 3.11 – 3.03 seconds. But if the entry flow is the same 500 pcu/hour and circulating flow doubled 1000 pcu/hour, the critical gap value decreases and falls in zone G between 2.80 - 2.41 seconds.

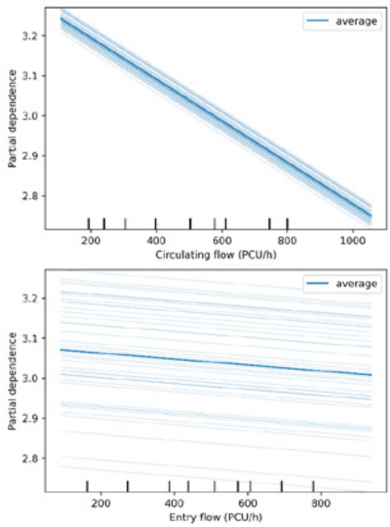

Fig. 14.
Partial Dependency Plots of Circulating and Entry Flow with Critical Gap

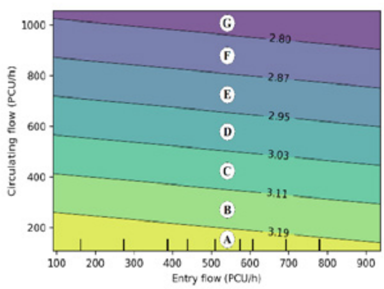

Fig. 15.
Partial Dependence Heatmap for Circulating and Entry Flow with the Critical Gap

5. Discussion, Conclusion and Future Work

The critical gap is a very important parameter that provides more information about the operation of roundabouts, and since the capacity of roundabouts depends on this value along with follow-up time in the calculation, a good estimation of this parameter is important. In this research Raff's method was implemented. After finding all the critical gaps, it has been noticed that there is a variation of results for each roundabout's entry, which led the author to investigate the reason behind these variations further. Thus, the investigation of different real data parameters such as circulating flow and entry flow was carried out to find the correlation between these parameters and the critical gap.

First, the coefficient of determination R^2 in (Fig. 12) presents a general idea about the relationship between the parameters and the critical gap. It was observed that the circulating flow parameter has a higher correlation with the critical gap as compared to entry flow. In order to enhance the correlation results, another statistical analysis has been performed, implementing the gradient boosting tree function. Better results have been obtained, as circulating flow showed a higher correlation with the critical gap and entry flow has got a meaningful correlation value (Fig. 13). Moreover, coefficient of determination R^2 of whole dataset was enhanced. More comprehensive representation is needed to have an insight understanding of these relationships, so partial dependence plots (Fig. 14) as well as partial dependence heatmap (Fig. 15) was generated.

In conclusion, it has been found that:

- Critical gap has a very low correlation with entry flow, meaning no matter how high the entry flow is, the critical gap slightly changes;
- Critical gap has a higher correlation with circulating flow, meaning that with the increase of the circulating flow, the critical gap value tends to decrease, due to the fact that the drivers tend to accept smaller gap;
- The correlation between entry flow and critical gap after data training was improved greatly;
- Partial dependence heatmap was generated. Different zones combining both parameters with the critical gap. The partial dependence heatmap gives a richer overview of this situation;
- By using only circulating and entry flow critical gap value can be obtained graphically. Which is more time efficient for traffic engineers to find critical gap value of a certain roundabout;

By taking the critical gap value using (Fig. 15) The Highway Capacity Manual (HCM, 2010) formula for roundabout capacity can be easily adjusted.

The methodology of this paper is transferable and can be used in other same driving culture. Future work is needed. More complicated roundabouts geometry, such as single-entry lane against two conflicting lanes, double-entry lane against one conflicting lane and turbo roundabouts, can be investigated also. Investigating the effect of velocity of circulating vehicles on the critical gap can be studied too.

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