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OPTIMIZING HIGHWAY PROFILES FOR INDIVIDUAL COST ITEMS

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Abstract: According to the current practice, vertical alignment of a highway segment is usually selected by creating a profile showing the actual ground surface and selecting initial and final grades to minimize the overall cut and fill quantities. Those grades are connected together with a parabolic curve. However, in many highway construction or rehabilitation projects, the cost of cut may be substantially different from that of fill (e.g. in extremely hard soils where blasting is needed to cut the soil). In that case, an optimization process will be needed to minimize the overall cost of cut and fill rather than to minimize their quantities. This paper proposes a nonlinear optimization model to select optimum vertical curve parameters based on individual cost items of cut and fill. The parameters selected by the optimization model include the initial grade, the final grade, the station and elevation of the point of vertical curvature (PVC), and the station and elevation of the point of vertical tangency (PVT). The model is flexible to include any design constraints for particular design problems. Different application examples are provided using the Evolutionary Algorithm in Microsoft Excel's Solver add-in. The application examples validated the model and demonstrated its advantage of minimizing the overall cost rather than minimizing the overall volume of cut and fill quantities.

Keywords: highway profiles, highway vertical alignment, nonlinear optimization, highway rehabilitation.

1. Introduction

Highway design is a complex process where conflicting design requirements should be satisfied at the same time (Dell'Acqua, 2012). A fundamental element in highway design is the design of vertical curves, which are used to connect initial grades, g_1, with final grades, g_2, as shown in Fig. 1. Highway vertical curves are usually parabolic with a constant rate-of-change of grade that changes the grade from g_1 to g_2 (AASHTO, 2010). In order to evaluate an existing highway vertical curve for rehabilitation projects, the following data will be needed (as shown in Fig. 1):

- The initial grade, g_1;
- The final grade, g_2;
- The location (station) of the point of vertical curvature (PVC), x_1;
- The location (station) of the point of vertical tangency (PVT), x_2;
- The elevation of the point of vertical curvature (PVC), y_1; and
- The elevation of the point of vertical tangency (PVT), y_2.

This paper examines the impact of different risk factors that may contribute to rollover collisions in order to help develop countermeasures that limit them. The factors investigated include driver-related,

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vehicle-related, environmental-related, and roadway-related factors. To avoid the bias that may be caused by interactions among different drivers, this analysis focuses on rollover related to single-vehicle collisions so that the behavior of the driver of the collided vehicle can be analyzed more effectively.

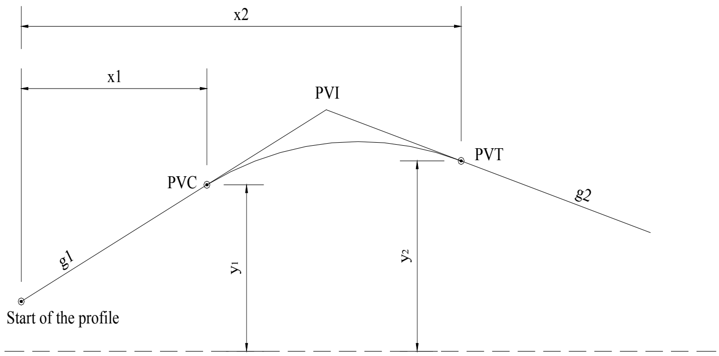

Fig. 1.
Characteristics of a Highway Vertical Curve

The above data may not always be available due to soil consolidation and prior resurfacing and maintenance projects for the highway. Based on that, the elevation of the highway centerline is usually measured at different stations and the resulting profile usually has an irregular shape that should be fitted into a highway vertical alignment by visually fitting two straight lines and a parabolic curve to the profile. This visual method is time consuming and may not be adequately accurate.

To overcome the limitation of the above visual method, an analytical method was developed (Easa et al., 1998) to accomplish two simultaneous tasks:

- To identify the start and end of linear (tangent) and curved segments automatically based on the trend of profile data; and
- To sequentially fit straight lines to the linear segments and clamped cubic spline functions to the curved segments.

The limitation of the above method is that it used third-degree spline lines (which have varying curvature and are more complex to analyze) to simulate a second - degree parabolic curve. The spline lines were used due to the fact that fitting vertical curves to the profile data using regression analysis (which by nature has no constraints) would not normally satisfy the constraints related to the connection between the tangents and the parabolic curve.

Another method was developed to present a linear optimization model (Easa, 1999), which allowed the use of constraints for fitting straight lines and parabolic curves to highway profile data. In that case both the tangents and parabolic curves were fitted simultaneously to the profile data, thus producing a better alignment than that of the sequential fitting. The only limitation in that method was that it considered locations (stations) of the start and end points of the parabolic curve (x_1 and x_2) to be known. That assumption was essential for linear optimization model to limit decision variables to four variables (g_1, g_2, y_1 and y_2) subject to linear constraints. That model was later extended (Easa, 2008) by utilizing a mathematical trick involving three binary variables to model the discontinuities at the start and end of the vertical curve, which ultimately resulted in convergence to the guaranteed globally optimum solution.

Another solution was introduced (Hu et al., 2004) to improve the above method by using least-square method instead of the linear programming optimization with assuming the start and end points of the parabolic curve (x_1 and x_2) to be decision variables; and therefore there would be six decision variables in total (g_1, g_2, y_1, y_2, x_1 and x_2).

In all the above methods, the objective function was to minimize the overall volume of cut and fill. However, in many situations, the unit cost of cut may be substantially different from the unit cost of fill. An example situation is where the soil is extremely hard so that blasting is needed to cut the soil. Based on that, this paper proposes a new nonlinear optimization model that considers different unit costs for cut and fill. In this case, the objective function is to minimize the overall cost of cut and fill, which is calculated as the volumes of cut and fill multiplied by the unit costs of cut and fill, respectively. Several application examples are provided to illustrate how the model is used in different situations.

2. Vertical Curve Equations

This given that g_1 and g_2 are the initial and final grades, respectively (positive for upgrade and negative for downgrade), and according to design guides (AASHTO, 2010), the algebraic difference in grades, A, for a vertical curve is given as Eq. (1):

$$A = g_2 - g_1 \tag{1}$$

If the length of the vertical curve is L, the rate of change of grade, r, is Eq. (2):

$$r = A / L \tag{2}$$

The inverse of the r value given above is the K value, which is the length of the vertical curve needed to effect 1% change in the slope of the vertical curve. Since highway vertical curves are usually equal-tangent, the point of vertical curvature (PVC) and the point of vertical tangency (PVT) are both located at equal distances from the point of vertical intersection (PVI). Assuming that the origin point is located at PVC, the offset, Y, of a point at distance x is given by Eq. (3):

$$Y = rx^2/2 \tag{3}$$

The corresponding elevation, y, is given by Eq. (4):

$$y = g_1 x + (rx^2/2) \tag{4}$$

By differentiating Eq. (4) with respect to x and equating dy/dx to zero, the location of the highest (or lowest) point, x_{hl}, can be shown to be equal to $(-g_1/r)$.

3. Optimization Model

For a highway profile consisting of two grades connected by a vertical parabolic curve, the station and elevation of any point, i, on the profile (x_i and y_i) can be computed from the coordinates of PVC and PVT that are (x_1, y_1) and (x_2, y_2), respectively. It is required to fit two tangents (grades) and a parabolic curve to the profile data. The range of the stations of the start and end of the parabolic curve (x_1, x_2) can be specified based on the shape of the profile. The number of data points of the initial grade, the parabolic curve, and the final grade are denoted by K, L, and M, respectively with total number N of data points. The exact locations of start and end points, (x_1, y_1) and (x_2, y_2), and the initial and final grades, g_1 and g_2, are all decision variables that need to be computed. For any point, i, the difference between the estimated and observed elevations can be given by Eq. (5):

$$d_i = y_{ei} - y_{oi} \tag{5}$$

where y_{ei} is the estimated elevation of point i and y_{oi} is the observed elevation of point i.

For the initial grade, the estimated elevation, y_{ei}, of a point that has station x_i is given by Eq. (6):

$$y_{ei} = y_1 - g_1(x_1 - x_i); \quad x_i \leq x_2 \tag{6}$$

For the parabolic curve, the offset and the corresponding curve elevation of a point (x_i, y_i), based on Eq. (3) and Eq. (4), are given by Eq. (7) and Eq. (8):

$$Y_i = r(x_i - x_1)^2 / 2 \tag{7}$$

$$y_{ei} = y_1 + g_1(x_i - x_1) + (g_2 - g_1)(x_i - x_1)^2 / 2(x_2 - x_1); \quad x_1 \leq x_i \leq x_2 \tag{8}$$

For the final grade, the elevation, y_{ei}, of a point that has station x_i is given by Eq. (9):

$$y_{ei} = y_2 + g_2(x_i - x_2); \; x_i \geq x_1 \tag{9}$$

If the elevations of the first and final points on the profile are both constraints, and to ensure linearity of the initial and final grades, the elevations of PVC and PVT may both be calculated according to the following equations (Eq. (10) and Eq. (11)):

$$y_1 = y_{o1} + g_1(x_1) \tag{10}$$

$$y_2 = y_{oN} + g_2(x_{oN} - x_2) \tag{11}$$

In the above equation, the parameter y_{o1} is the elevation of the first point on the profile, and the parameters x_{oN} and y_{oN} are the station and elevation of the last point on the profile.

The positive values for the difference, d_i, indicate fill sections, and the negative values indicate cut sections. Let the variables w, s_f, and s_c denote the road width, side slope at fill sections, and side slope at cut sections, respectively. Based on that, the total fill and cut volumes are calculated using the following equations (where V_f is the total fill volume and V_c is the total cut volume) (Eq. (12) and Eq. (13)):

$$V_f = w \sum_{i=1}^{n} d_i + s_f \sum_{i=1}^{n} (d_i)^2 \; [where \; d_i > 0] \tag{12}$$

$$V_c = w \sum_{i=1}^{n} d_i + s_c \sum_{i=1}^{n} (d_i)^2 \; [where \; d_i < 0] \tag{13}$$

If the unit costs of fill and cut are C_f and C_c, respectively, the total costs of fill and cut are calculated using the following equations (where TC_f is the total cost of fill and TC_c is the total cost of cut) (Eq. (14) and Eq. (15)):

$$TC_f = V_f \times C_f \tag{14}$$

$$TC_c = V_c \times C_c \tag{15}$$

The objective function is to minimize the sum of the total cost of fill and total cost of cut (Eq. (16)):

$$Minimize \left[TC_f + TC_c\right] \tag{16}$$

To ensure an equal-tangent vertical curve, the following constraint must be added (Eq. (17)):

$$g_1(L/2) + g_2(L/2) = y_2 - y_1 \quad (17)$$

Knowing that $L = x_2 - x_1$ and by re-arranging the Eq. (17), it can be reduced to Eq. (18):

$$2(y_2 - y_1) - (g_1 + g_2)(x_2 - x_1) = 0 \quad (18)$$

The model is also subject to the following constraints (Eq. (19) and Eq. (20)):

$$x_1 > x_{o1} \quad (19)$$

$$x_2 < x_{oN} \quad (20)$$

In the above equation, the parameter x_{o1} is the station of the first point on the profile. To ensure adequate sight distance on the designed vertical curve, the following constraint must also be met (Eq. (21)):

$$(x_2 - x_1)/|g_2 - g_1| > K_{design} \quad (21)$$

In the above equation, the parameter K_{design} is the minimum K value required for the vertical curve to ensure adequate sight distance, which depends on the design speed, as given by geometric design guides (e.g. AASHTO, 2010). To ensure that all decision variables have non-negative values, g_1 and g_2 may be replaced by two non-negative variables (Eq. (22) and Eq. (23)), such that:

$$g_1 = g_{11} - g_{12} \quad (22)$$

$$g_2 = g_{21} - g_{22} \quad (23)$$

Eqs. (14-23) represent the nonlinear optimization model, which may be solved using any commercially available optimization software. More constraints may be added for particular vertical curve problems such as maintaining a certain height above an underpass or below an overpass or maintaining a certain elevation at a certain station for intersection with another roadway.

4. Application Examples

A hypothetical example is provided to validate the developed nonlinear optimization model. In the example, the given profile data typically represent that shown in Fig. 2 without any error; and therefore the optimization value is expected to be zero. The elevations at different stations were calculated based on the parameters shown in Table 1.

The example was solved using the Evolutionary Algorithm in Microsoft Excel's Solver add-in. The decision variables calculated by the software were found to precisely match those shown in Table 1 with the optimization value found to equal zero, which validates the developed model.

The same example was solved again after the profile data have been deliberately altered to reflect a real-world situation where vertical curve elevations are changed as a result of soil consolidation and prior resurfacing projects. The example was once solved with assuming the unit costs of cut and fill to be \$50/m^3 and \$30/m^3, respectively. In that case, the feasible solution found by the optimization model calculated the quantities of cut and fill as 1.31 m^3 and 2.77 m^3, respectively. The total cost of cut and fill was \$148.67. The example was solved again with assuming an extreme case of high unit cost of cut at \$3000/m^3 with keeping the unit cost of fill as \$30/m^3. In that case, the feasible solution was different and the quantities of cut and fill were 0.02 m^3 and 1.90 m^3, respectively, with total cost of cut and fill $1930.57. This substantial difference in the cut volume between the two examples is a result of the high unit cost of cut in the latter example, which demonstrates the advantage of using the developed model to minimize the overall cost rather than minimizing the overall cut and fill volumes. This objective is different from the objective of the previously developed models discussed in this paper that focused on minimizing the overall cut and fill volume. The profiles of the original data and the two examples are shown in Fig. 3 for comparison purpose.

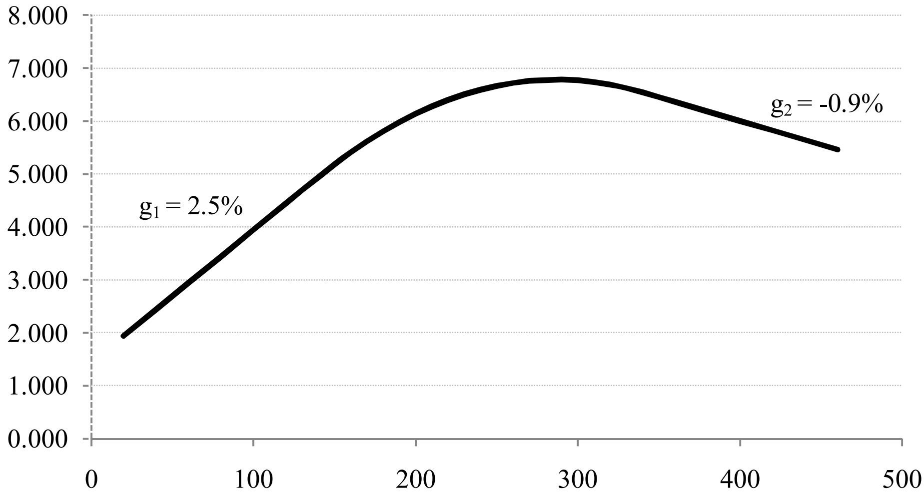

Fig. 2.
Profile of the Validation Example

Table 1
Design Parameters and Decision Variables for the Validation Example

Parameter	Value
Design parameters:	
Width of roadbed (W)	14
Fill side slope	3
Cut side slope	1
Decision variables:	
Initial grade (g_1)	2.5%
Final grade (g_2)	-0.9%
Station of PVC (x_1)	140
Station of PVT (x_2)	340
Elevation of PVC (y_1)	4.947
Elevation of PVT (y_2)	6.547

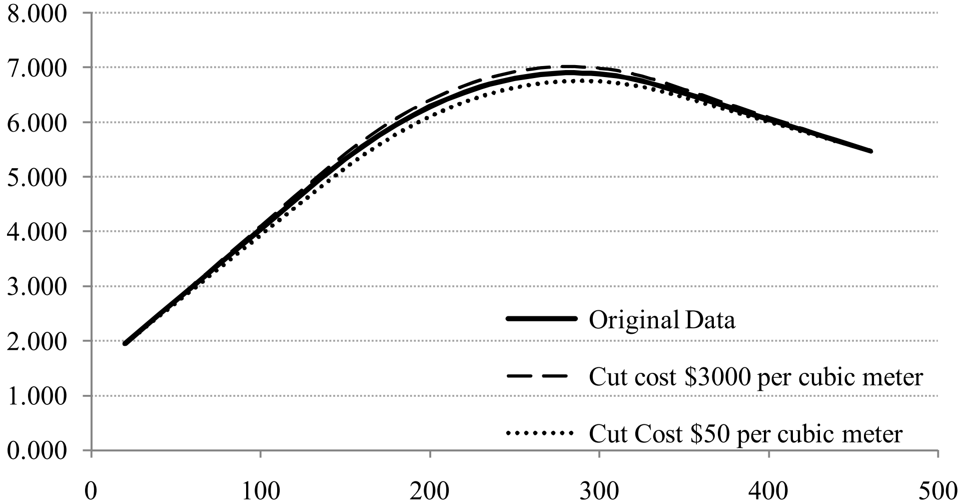

Fig. 3.
Profiles of the Original Data and the Two Application Examples

5. Conclusion

In this paper, a nonlinear optimization model was developed to select optimum vertical curve parameters for rehabilitation projects based on individual cut and fill cost items. The objective of the developed optimization model is to minimize the overall cut and fill costs rather than minimizing their overall quantities. The parameters selected by the optimization model include the initial grade, the final grade, the station and elevation of the point of vertical curvature (PVC), and the station and elevation of the point of vertical tangency (PVT). The model has the flexibility to include any constraints needed for particular design problems such as setting certain stations at certain elevations. The model also has the flexibility to include more specific cost itemization such as selecting different unit costs for different cut depths. Different application examples were provided using the Evolutionary Algorithm in Microsoft Excel's Solver add-in, which validated the model and demonstrated its advantage of minimizing the overall cost rather than minimizing the overall cut and fill volumes. Although the model is designed for rehabilitation projects with a single parabolic curve, it can be extended to optimize more-complex profiles with multiple curves. The developed model can also be applied for designing new profiles by setting the appropriate design constraints.

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