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THE VEHICLE ROUTING PROBLEM WITH LIMITED VEHICLE CAPACITIES

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Abstract: The vehicle routing problem (VRP) has been an important research topic during the last decades because of his vital role in the productive systems efficiency. Most of the work done in this area has been directed to solve large scale problems which may not apply for small companies which are a very important engine of the world economy. This paper approaches the problem when limited vehicle resources are present and road transportation is used. This study assumes variable customer orders. Variable volume and weight vehicle capacities are considered and the proposed algorithm develops the vehicle delivery routes and the set of customer orders to deliver per vehicle minimizing a cost objective function. In sampling small company's logistics costs, big cost savings are found when using the proposed method.

Keywords: vehicle routing, mathematical programming, tabu search, cut cost.

1. Introduction

The VRP was introduced in 1959 by Dantzig and Ramser (1959) who described the problem as gas delivery to gas stations and five years later Clarke and Wrigth (1964) developed the first algorithm to solve it. Since then thousands of papers have been written related to the VRP. Yeun et al. (2008) and Laporte (1992) made a comprehensive review of models and solutions related to the VRP and its variants and conclude that exact algorithms can only solve relatively small problems and a number of approximate algorithms have proved very satisfactory results for large problems.

Approximate algorithms as tabu search methods, simulated annealing and ant colony optimization have been used by researchers to solve the VRP (Chuin et al., 2003; Tang Montané and Galvao, 2006; Gendreau et al., 1994; Bell and McMullen, 2004).

Evolutionary algorithms as the genetic algorithm have been used by Chang and Chen (2007). Assuming unlimited vehicle capacities, encouraging results were reported. Baker and Ayechew (2003) also used a genetic algorithm assuming a specific weight of goods to be delivered and Marinakis and Marinaki (2010) proposes a genetic algorithm with an improvement phase assisted by knowledge stored within the parent solutions.

Tabu search is mainly recognized as the algorithm with better results in terms of quality of solution and computation time (Daza et al., 2009).

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Vehicle routing problems found in the literature are usually hard to apply in real life situations. Most research is aimed to solve the biggest possible problem in minimum computation time and in order to achieve it some variables are not taken into account. Some researchers have made efforts to develop models and solution methods for real life situations (Goel and Gruhn, 2008). They propose a rich vehicle routing problem incorporating various complexities found in real life situations, using a large neighborhood search method for solution.

In this research we target a specific situation which is shared by many small and medium companies which is not taken into account in the traditional VRP and its variants.

A set of customer orders are due for delivery from a depot with known individual weight and volume. The company has a set of vehicles for delivery with variable weight and volume capacities; we want to find the best possible assignment of orders for each vehicle and its routes satisfying vehicles capacities in order to minimize a cost objective function. In order to improve logistics efficiency, managers may enter the set of customer orders to deliver with individual weight and volume per order and the model will find which orders go to each vehicle and the routes for each vehicle in order to minimize a cost objective function.

After sampling small businesses, the volume constraint was more active than the weight constraint due to the use of small vehicles as motorcycles, automobiles and trucks.

In the following section 2 we state the model formulation given a set of assumptions. In section 3 we propose a method of solution to our model and define parameters, variables and a flow diagram of the algorithm. In section 4 we apply the model to improve the logistics of a company that distributes a variety of foods in Mexico. In section 5 we present the conclusion of the study.

2. Model Formulation

Various mathematical formulations for the VRP have been published (Daza et al., 2009; Tang Montané and Galvao, 2006; Calvete et al., 2004). In this research we adapted previous models to the situation described in the previous section and using the following assumptions.

Let $G=(A,C)$ be an undirected graph, where $A=\{0,1,.....,n\}$ is a set of nodes or vertices where $\{1,.....,n\}$ represent the customers and node $\{0\}$ represent the depot.

The VRP may be defined as the design of a set of routes for the available vehicles such that the following conditions are met:

a) Every customer is visited only once for one vehicle.
b) All vehicle routes begin and end at the depot.
c) All constraints are satisfied.

Let C be a set of arcs, $arc(i,j)$, $i \neq j$ where every arc has an associated cost c_{ij}; $i,j=0,1,.....,n$.

If customers and depot are located at points (a_i,b_i) ; $i=0,1,.....,n$ and we use road transportation, rectilinear distance becomes a good estimate of the distance between two points then Eq. (1)

$$d_{ij} = |a_i - a_j| + |b_i - b_j| \quad (1)$$

represents the rectilinear distance between points (a_i, b_i), and (a_j, b_j).

Let p_i be the weight of individual customer orders in kilograms; $i=0,1,.....,n$; and v_i the volume of individual customer orders; $i=0,1,.....,n$; in addition let m be the number of vehicles and cp_k ; $k=0,1,.....,m$; be the weight capacity of each vehicle; and cv_k ; $k=0,1,.....,m$; the volume capacity of each available vehicle.

Let $X=(x_{ijk})$ be the decision variable matrix, where $x_{ijk}=1$ if the arc (i,j) is covered by vehicle k, and $x_{ijk}=0$ if the arc (i,j) is not covered by vehicle k therefore $x_{ijk} \in \{0,1\}$.

Our problem may be defined as follows:

$$Min \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} d_{ij}\, x_{ijk} \quad (2)$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{m} x_{ijk} \le m; \quad i=0 \quad (3)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{m} x_{ijk} = 1 \quad ; \; i=1,\dots,n \quad (4)$$

$$\sum_{j=1}^{n} x_{ijk} = \sum_{i=1}^{n} x_{ijk} \; ; \; i,j=1,\dots,n \quad (5)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} p_i\, x_{ijk} \le cp_k \; ; k=1,\dots,m \quad (6)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} v_i\, x_{ijk} \le cv_k \; ; k=1,\dots,m \quad (7)$$

Let S be any subset of $A - \{0\}$ then:

$$\sum_{i\in S}^{n}\sum_{j\in S}^{n} x_{ijk} \le |S|-1 \; ; \forall S \subseteq (A-\{0\}) \quad (8)$$

$$|S| \ge 2\; ; \; i,j = 1,\dots,n$$

$$k=1,\dots,m$$

$$x_{ijk} \in \{0,1\} \quad (9)$$

The aim of our problem is to find the matrix $X=(x_{ijk})$ size $n \times n \times m$ where the binary variables x_{ijk} indicate which arcs (i,j) are to be visited by vehicle k while satisfying the weight and volume capacities constraints.

The objective function (Eq. (2)) minimizes the routing rectilinear distance. We may replace d_{ij} with c_{ij} in Eq. (2) in order to add more routing cost variables. Constraint (3) ensures that no more thanvehicles are used for delivery. Constraints (4) and (5) are required to make sure only one vehicle visits every customer. Constraint (6) ensures that weight capacities of the vehicles are met. Constraint (7) is similar to constraint (6) in order to fulfill the volume capacities. At last, constraint (8) ensures that all routes are connected and constraint (9) defines all decision variables as binary.

When business employ different vehicles to deliver orders they naturally have different levels of energy consumption and maintenance costs, therefore, we may define $c_{ij} = ev_j * d_{ij}$, where ev_j the efficiency of vehicle is j. The range of ev_j is defined as:

$$0 \le ev_j \le 1$$

Then Eq. (2) may be replaced by:

$$Min \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} c_{ij}\, x_{ijk} = Min \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} ev_j\, d_{ij} x_{ijk} \tag{10}$$

3. Method of Solution

Since the model is a hard combinatorial problem it is doubtful that exact methods or software as CPLEX can find optimal solutions in reasonable computation times. We decided to use Tabu search in one stage of the procedure.

Tabu search is a technique which incorporates adaptive memory and responsive exploration. The adaptive memory allows the implementation of procedures capable of searching the solution space economically and effectively. Responsive exploration allows exploiting good solution features and exploring new promising regions.

Tabu search tries to avoid local optimum by using short and long range memories and aspiration criterion. In this technique solutions jump from one solution to the best neighboring solution not taking into account if the new solution does not improve the former solution.

In this study we used two types of aspiration criterion. Aspiration by defect, when all possible movements are classified as tabu, then we select "the least tabu movement", and secondly aspiration by objective, when a movement produces the best solution to that stage, then it is not classified as tabu.

The heuristic methods most used for the VRP may be classified in four categories as follows:

a) Constructive methods. These methods add customers to the vehicle routes one by one under a specified criterion, for example Clarke and Wrigth (1964);
b) Grouping first then routing. A set of customers are divided in subsets. One vehicle is assigned to each and every subset is solved independently, for example Gillett and Miller (1974);
c) Routing first then grouping. In this case a traveling salesman problem is solved for all customers then the route is partitioned in segments, one for each vehicle, for example Gendreau et al. (1994);
d) Improvement methods. These methods usually have an initial phase where an initial feasible solution is found, then in later stages the initial solution is improved using some defined method, for example Daza et al. (2009).

The methodology that we use in this research is a combination of routing first then grouping and the improvement methods. We did not used the method of grouping first then routing as that approach may not give you a good result when the depot is not centered with respect to the set of customers. We divided the procedure in four phases:

Phase I. Initial feasible solution. Given a set of customers for delivery we start the route at the depot and choose as first customer the one closest to the depot using rectilinear distance, Eq. (1). Then, among the rest of the customers we define as customer two the one closest to customer one and so on. Therefore, we end up with a routing order for the whole set of customers. To finish phase I, we assign vehicles to orders according to vehicles volume and weight capacities and assigning to each vehicle as many orders as possible in order to minimize the number of vehicles used.

Phase II. In this phase we make interchanges on the delivery order of each subset, that is, inside the delivery route of each vehicle for all possible combinations.

Phase III. In this phase we use the meta-heuristic procedure tabu search to look for possible improvements interchanging customer orders among vehicles while maintaining the capacities limitations.

Phase IV. When vehicles used for delivery have different capacities and efficiencies, the order in which vehicles are assigned to routes have a strong impact on the total cost, therefore, in this phase we repeat phases I to III for all possible permutations for the order in which vehicles are assigned, that is m! or the number of vehicles with different characteristics. Vehicles with equal capacities and efficiencies do not need to be included.

The proposed algorithm was developed in MATLAB R2010a version 7.10.0.499 using the following parameters and variables:

n = number of customers to deliver
m = number of vehicles
C = matrix (nx2) costumer coordinates
P = vector (n) individual orders weight in kilograms
V = vector (n) individual orders volume in cubic meters
VE = vector (m) vehicles efficiency
CPI = vector (m) vehicles weight capacity in kilograms
$CV\,I$= vector (m) vehicles volume capacity in cubic meters.

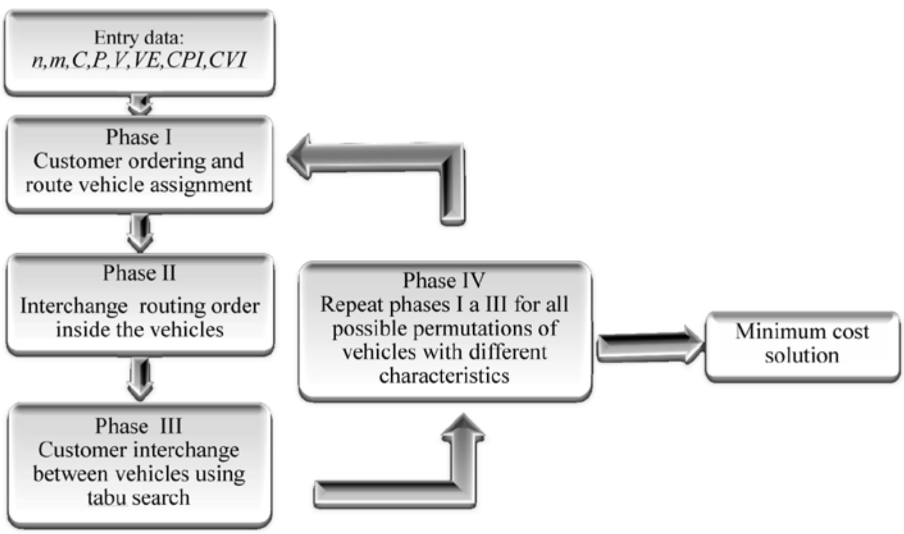

Fig. 1.
Flow Diagram for the Proposed Algorithm

The depot is located in point (0,0) and all distances are measured using rectilinear distances (Eq. (1)). A flow diagram of the proposed algorithm is shown in Fig. 1.

4. Results

The model was used in a company which distributes a variety of foods in the Querétaro city area, México. The company sells a great variety of foods with differ in weight and volume, in addition some require temperature control. The company has 3 vehicles with different capacities and efficiencies which are shown in Table 1. A vehicle efficiency is measured in a scale from 0 to 1, where 1 means ideal vehicle efficiency in terms of energy consumption and maintenance.

Table 1
Vehicle's Efficiency, Weight and Volume Capacity

Concept	Vehicle		
	1	2	3
Weight capacity (kilograms)	500	200	75
Volume capacity (cubic meters)	2.5	0.8	0.2
Efficiency	0.55	0.65	0.95

Table 2 shows customer's locations when the depot is located at point (0,0), weight and volume of individual orders.

Table 2
Customer's Location, Orders Weight and Volume

Customer number	x-coordinate	y-coordinate	Order weight (kilograms)	Order volume (cubic meters)
01	-2.1	-5.2	2.2	0.06
02	-3.1	-6.2	4.5	0.11
03	-8.8	-18.6	10.9	0.14
04	2.6	-25.2	12.2	0.19
05	0.8	-13.5	2.5	0.05
06	-1.1	-10.5	14.3	0.22
07	-1.2	-32.3	3.8	0.08
08	-8.5	-18.8	5.8	0.06
09	1.7	-21.2	4.4	0.09
10	0.5	-13.3	1.5	0.02
11	-3.0	-6.5	8.5	0.23
12	-0.9	-35.2	19.7	0.29
13	-8.3	-18.2	9.7	0.18
14	1.5	-20.7	4.0	0.19
15	10.3	-30.5	5.5	0.14
16	0.7	-14.6	6.2	0.12
17	-0.8	-30.2	4.9	0.60
18	-0.5	-12.8	12.5	0.29
19	2.8	-24.3	3.2	0.06
20	-0.3	-6.5	5.5	0.19

A company's typical route is shown in Table 3.

Table 3
A Typical Vehicle Scheduling and Routing

Vehicle	Routing assignment
1	16-10-18-13-03-14-09-19-04-17-07-12-15
2	11-02-20-06
3	01-05-08

Vehicles start at the depot, follow the assigned route and end at the depot at all times. This procedure is followed as a policy by most companies. The total cost and distances traveled by vehicles for the assignment shown in Table 3 are presented in Table 4. The proposed algorithm was programmed using MATLAB R2010a in a laptop with an INTEL COREi5 processor.

Table 4
Distance Run by Vehicles and Total Cost for Table 3 Assignment

Vehicle	Distance (kilometers)	Vehicle volume capacity (cubic meters)	Vehicle weight capacity (kilograms)	Vehicle efficiency
1	121.6	2.5	500	0.55
2	27.8	0.8	200	0.65
3	60.4	0.2	75	0.95
Total distance	209.8			
Total cost	142.33			

Table 5
Vehicle Scheduling and Routing Assignment Using the Proposed Algorithm

Vehicle	Routing assignment
1	18-16-14-19-04-17-07-12-15-08-03-13
2	09-05-10-06-11-02-01
3	20

Table 6
Distance Traveled by Vehicles and Total Cost Comparison

	Typical assignment	Proposed assignment
Vehicle 1 traveled distance (kilometers)	121.6	117.6
Vehicle 2 traveled distance (kilometers)	27.8	52
Vehicle 3 traveled distance (kilometers)	60.4	13.6
Total distance (kilometers)	209.8	183.2
Total cost Eq. (10)	$142.33	$111.40

Using the proposed algorithm, we find the assignment shown in Table 5.

Table 6 makes a distance cost comparison between the typical assignment and the assignment of the proposed algorithm.

The proposed assignment total cost is 21.74% lower than the typical assignment total cost and the total distance traveled by the vehicles using the proposed assignment is 12.68% lower than the distance traveled using the typical assignment.

This savings may have an important impact in companies which have to deliver frequently its products to several customers.

5. Conclusions

A huge amount of research has been done in the last decades for the VRP, however, it is usually difficult to apply in small companies as researchers aim to solve the biggest possible problem with the smallest computation time and therefore making assumptions which differ from real situations. This paper shows a simple procedure to obtain a cost efficient routing schedule using any popular laptop, the input required is vehicle capacities, vehicles efficiency, set of customers to deliver and weight and volume of individual orders. The output is the delivery order of each vehicle.

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